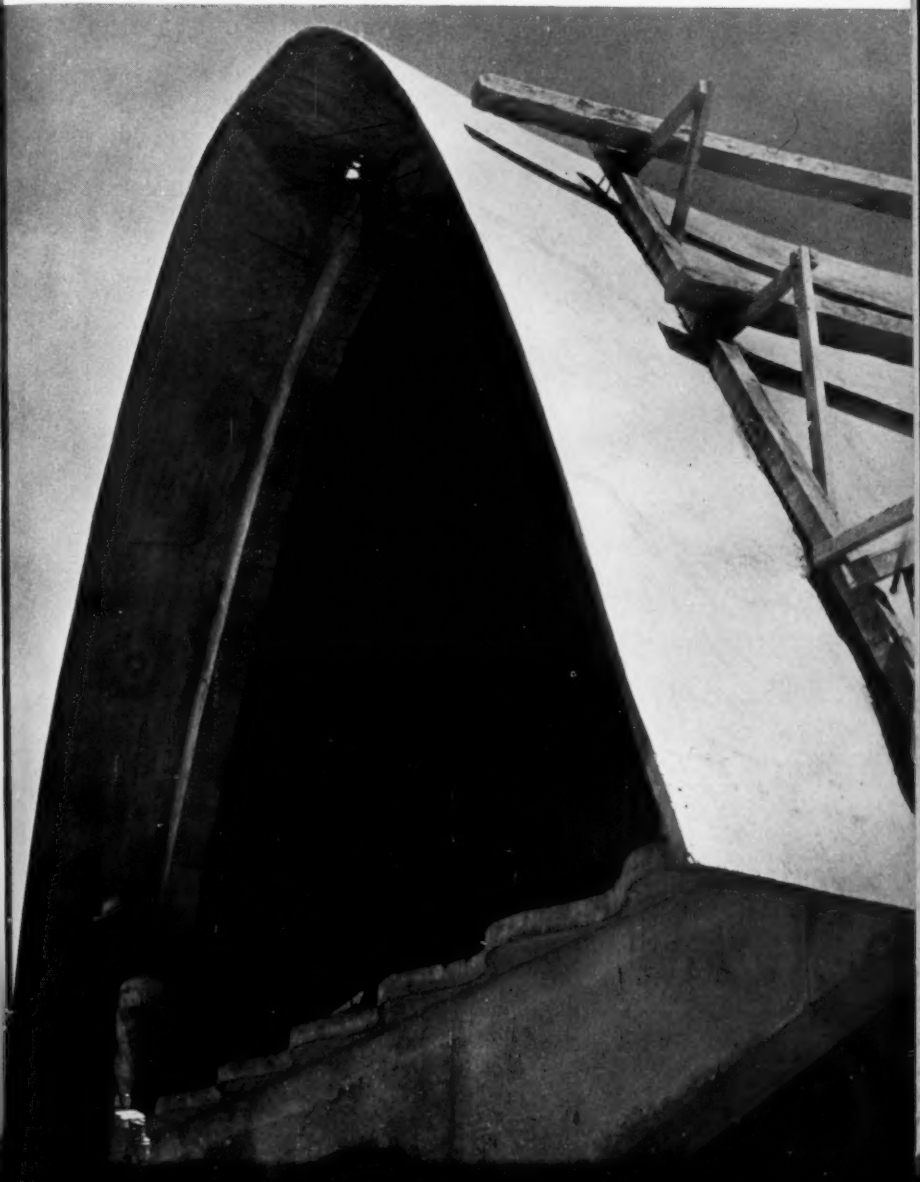
ACI News Letter

Vol. 23 No. 4 JOURNAL of the AMERICAN CONCRETE INSTITUTE December 1951







Filling pier form with coarse aggregate. This mass, surrounding the steel H-piles, was then Intrusion-grouted to form the Prepakt Concrete pier. Due to special characteristics of Intrusion grout, water in the cofferdam was driven out by the rising grout pumped in from the bottom of the form.

Finished Prepakt H-pile concrete pier showing above-water design and relative size. Note steel sheeting which was originally used as a cofferdam and concrete form, and retained as part of final structure.

PREPAKT PROVIDES LOW COST DEEP PIERS

Another case where Prepakt Concrete provided great practical economy is found in the construction of four deep piers (as much as 130 feet) for a northern bridge.

Each of the four piers, identical in measurement and design, consisted of 33—12 x 12 H-piles driven to rock or refúsal, then encased in a mass of Prepakt concrete. The Prepakt methods and materials used resulted in several outstanding economies.

First, the construction required no caissons, dewatering, or other costly methods common to underwater concrete placement. In addition, the Prepakt H-pier design required only 50% of the concrete necessary for a comparable pier of standard design. Also, the design required far less piles than the usual number for other type piers and provided a much faster construction schedule.

These advantages plus Prepakt high strength and extreme durability are some of the reasons for the rapid increase in use of Prepakt for concrete structural projects requiring the best.

Full information on Prepakt as used in this and similar structures may be had by writing or calling the Main Office, Cleveland, Ohio.

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ACI

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December 1951

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St. Louis Regional Meeting Huge Success

229 register for event Oct. 30 and 31, 1951

Hospitable ACI friends made the Regional Meeting one to be remembered. A city widely known for its civic activity seemed to outdo itself in welcoming ACI members, their wives and guests. A strong technical program of three sessions was supplemented on Tuesday by a luncheon in the Crystal Room of the Sheraton Hotel for the men; a luncheon and style show at the Park Plaza Hotel for the ladies followed by a tour of St. Louis; and a cocktail hour and dinner—the social highlight of the meeting. An inspection trip, which will be described later, ended the meeting on Wednesday afternoon.

If there had been a theme for the meeting it might have been, "Steel's Role in Concrete Construction," as most of the papers were pointed toward the use of steel in or with concrete.

The Regional Meeting Committee is to be complimented on excellence of the program and other arrangements.

Past President Robert F. Blanks, in speaking for the Board of Direction at the last session, said, "I think every one will agree that I have been given an impossible job of officially, on behalf of the American Concrete Institute, thanking the folks in St. Louis for what they have done for us. We have always heard about southern hospitality and western hospitality and New England hospitality, but I am sure from hereon we shall add another and call it St. Louisan hospitality. All my life I have heard an old song, to which I never attached much significance, except that it was a catchy tune that stuck in my head, and I would hear myself humming it ever so often. Now I am sure we all know what inspired that song, 'Meet Me In St. Louie.'

Continued on p. 4

"We may have come here unnourished, but I am sure we all agree that we are going away well fed, morally, mentally and physically. The caliber of food for thought that we have had here, as well as food for the body, has been something to remember for a good, long time.

"Again, all I can say is that there are certain things outside the realm of human endeavor, and one of them is to adequately thank the kind of folks who live in St. Louis, for what they have done for the folks who came to visit them."

FIRST TECHNICAL SESSION

ACI President Harry F. Thomson opened the meeting with a welcome to members and guests and expressed his great pleasure in presiding at a meeting of ACI in his home town. After reciting the history of regional meetings and announcing that the next one would be held in Chicago, Sept. 11, 12, 1952, at the time of the Centennial of Engineering, he turned the meeting over to Past President Frank H. Jackson.

Continuously reinforced concrete pavement

Mr. Jackson reviewed briefly the history of the speaker's subject and then introduced J. D. Lindsay, engineer of materials, Illinois Division of Highways, who presented "Four-Year Report on the Experimental Continuously Reinforced Concrete Pavement in Illinois." Mr. Lindsay's paper paralleled closely the three-year report made at the 1950 annual meeting of the Highway Research Board with the benefit of an additional year's observations. He reported that while the pavement is not old enough to justify definite conclusions as to ultimate performance present indications are that a properly designed continuously reinforced concrete pavement should give excellent service.

SECOND TECHNICAL SESSION

The second technical session, at which President Thomson presided, opened about a half-hour late due to the delay resulting in setting up a larger room than originally planned to take care of the unexpected crowd. Two hundred or more heard four excellent papers.

Malpractices on small jobs

Prof. A. W. Brust, Dept. of Civil Engineering, Washington Univ., St. Louis, reported on personal observations of variations in properties and behavior of concrete during 1947-1950 in a paper entitled "Ready-Mix Concrete Practices in the St. Louis Area." As in other large metropolitan areas, St. Louis faces the problem of getting satisfactory concrete for small jobs. While he cited instances of good work, he gave emphasis to malpractices which were shocking indeed. Professor Brust believed that there is a great need for extensive education at the elementary level all along the line from the specification writer and supervising architect to the foreman and concrete finisher. He stated that ACI had done an outstanding job of improving the quality of concrete in major structures where adequate specifications, inspection and testing are an integral part of the contract, and that perhaps it could do an even more outstanding job of education and improvement in the small job field which in the past too often has functioned without specifications or inspection. He closed with an appropriate quotation by John Ruskin: "It's unwise to pay too much, but it's unwise to pay too little. When you pay too much you lose a little money, that is all. When you pay too little, you sometimes lose everything, because the thing you bought was incapable of doing the thing you bought it to do. The common law of business balance prohibits paying a little and getting a lot. It can't be done. If you deal with the lowest bidder, it's well to add something for the risk you run. And if you do that, you will have enough to pay for something better."

Floor discussion indicated a consensus that a solution to the problem might be found if the concrete producer, architects, engineers, contractors, labor men and owners could get together with the objective of improvement of concrete practices.

Stanton Walker, director of engineering, National Ready Mixed Concrete Assn., remarked that Professor Brust had described malpractices in concrete that have existed for years and that they will be found not only in ready-mixed concrete but in much of the concrete on small jobs which do not have adequate supervision and control. He

Continued on p. 6

THANK YOU*

The American Concrete Institute is fortunate, indeed, in having members and friends in the St. Louis area devote their time, their energy, their talents and their leadership to the development, within the period of a few months, of a praiseworthy Regional Meeting.

It is fitting that the Board of Direction express its appreciation to the committee for a task well done and to the sponsors whose generous support made it possible.

Committee

A. Carl Weber, *Chairman*
Edward Asche, *Treasurer*
Clarence H. Ax, *Secretary*
William C. E. Becker, *Meeting Chairman*
Sidney Bierman, *Reservations*
Walter Bryan *Registration*
Carl Chappell, *Publicity*
Howard Coleman, *Inspection Trip*
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Thomas J. Skinker
G. L. Tarlton Contracting Co.

*Resolution adopted by the ACI Board of Direction, Oct. 31, 1951



ACI officials in St. Louis for the fall regional meeting inspect the first application on a large concrete frame building of Cofar, combined reinforcement and form for concrete. The building is the new 130,000 sq ft warehouse of Associated Grocers in St. Louis. Left to right are: Harry F. Thomson, Chicago, President; Frank H. Jackson, Washington, D.C., Past President; H. J. Gilkey, Ames, Iowa, Past President; and Albert H. Baum, Building Commissioner of St. Louis. MacDonald Construction Co. is the contractor and Sverdrup & Parcel, the engineers.

said that, without defending it in any degree whatever, he suspected the concrete described represented better concrete than that which was used for the same purpose before ready-mixed concrete came into the field.

He believed that Professor Brust had not made a very serious condemnation of the ready-mixed concrete producers themselves, but rather had seriously condemned the purchaser who used the concrete. The producer, he said, is in the business of selling the customer the kind of concrete he asks for and in trying to educate him as to the kind of concrete he should ask for. As long as the customer insists on buying concrete with only three or four sacks of cement in it so that it will be cheap, and with lots of water so that it will flow into the forms without manipulation, the type of work described will continue.

Steel forms

"Advantages of Steel Forms for Industrial Concrete Construction" was the title of an excellent construction paper by L. A. Wagner, director of engineering and research, Missouri Portland Cement Co. The use of all steel forms in the construction of large slurry tanks, walls and buildings proved to be economical and offered many advantages. Photographs showed the attractive clean-cut appearance in the finished structures. During construction the job was kept much cleaner and neater due to the absence of the clutter frequently associated with wood forms after they were removed and awaited cleaning for reuse, dismantling for salvage, or burning.

Chief advantages listed by the speaker were (1) strength, (2) rigidity, (3) speed of erection and dismantling and (4) appearance of the finished structure.

Testing welded wire fabric

Prof. E. W. Carlton and Mr. J. H. Senne of the Dept. of Civil Engineering, Missouri School of Mines and Metallurgy, Rolla, together presented a paper entitled, "Instrumentation and Strain Measurements in Welded Wire Fabric Reinforced Concrete Slab." They reported that although welded wire fabric has been used for reinforcement for over forty years, little was done to determine its properties as a structural material during the first half of that period. The coordinated research program described was sponsored by the Wire Reinforcement Institute and iritial work was done in 1948 by E. A. Weinel. His tests were so conducted that there was no bond between the concrete and the longitudinal wire and only anchorage of the weld would be involved. He concluded that longitudinal wires did not slip until the welds failed in shear and good welds would give an anchorage of about 90 percent of tensile strength of the wire.

In 1949 A. A. Becker conducted a second series of tests, the results of which indicated that the combination of longitudinal wire bond and weld anchorage was considerably greater than the strength of the longitudinal wires.



Of outstanding interest on the inspection trip was the stop at "Commission Row" with its 300,000 sq ft of concrete floor and 3.3 miles of precast wall panels shown cast 2-high in the foreground



Nobody stood under this panel as it was lifted and placed in 10 minutes or less

Photos courtesy Rock Products

A third series of tests by F. R. Heartz attempted to determine crack control properties of welded wire fabric.

All these investigations showed a wide variation in strength of welds and lead to the development of a weld tester by Professor Carlton and Prof. A. U. Kilpatrick which is reported as now being used by member companies of the Wire Reinforcement Institute as a production control test.

In addition to developing the tester, Prof. Carlton reported the following results from the research program described: Initiation of further work toward preparation of an ASTM Standard for testing welded wire fabric; development of a technique for placing SR-4 gages on small diameter wire; evidence that the average shear value of welded wire fabric exceeds the usable working stresses of the longitudinal wires; and evidence that closely-spaced small transverse wires can control transverse cracking.

Useful data

Raymond C. Reese in ten minutes described the results of seven years' work in preparing a handbook for engineers which is intended to make the selection and design of economical reinforced concrete structures almost as simple as those of steel. The book, which will be called "CRSI Design Handbook," and will be published by the Concrete Reinforcing Steel Institute at a time and price to be announced later,* and will contain tables and charts for the design of practically every type of beam, column, slab, wall, panel, etc., for almost every reasonable loading condition. The aim of the book, Mr. Reese said, is to take some of the drudgery out of concrete and let the engineer devote some of his talents to the

*Ed. note: When available, complete information will be published in the JOURNAL in both the *Current Reviews* and News Letter sections.



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economics and the satisfactoriness of his designs and to put his effort where it belongs. —"And," President Thomson added, "possibly the average golf score of engineers in the next ten years will be reduced as a result."

THIRD TECHNICAL SESSION

Multiple-story concrete tanks

Past President H. J. Gilkey presided and lost no time in introducing the first speaker, E. J. Critzas, construction engineer and vice-president of the Borsari Tank Corporation of America.

Mr. Critzas' paper, "Field Experiences in Building Multiple Story Reinforced Concrete Tanks," described briefly and illustrated with photographs the construction of tanks for the Anheuser-Busch Brewery in St. Louis. The tanks are rectangular and are used for the fermentation and storage of beer. They vary in width from 12 to 15 ft, in height from 7 ft 6 in. to 9 ft, and in length from 40 to 60 ft. Arranged in blocks of 2 or 3 per floor, they were built from 3 to 6 stories high and designed as multiple-story rectangular frames with the ceiling of the lower tier forming the floor slab of the upper. One type was designed for a liquid load only and another to withstand an internal gas pressure of 15 psi. All were lined with a special jointless ⅔-in. thick lining to protect the concrete from the chemical action of the contents and to provide a smooth, sanitary, odorless and tasteless surface. On the latest installation all longitudinal walls were to be prestressed to provide approximately 100 psi compression in the concrete to eliminate shrinkage cracks. The installation is now under observation to determine its effectiveness.

Prestressed concrete

Prof. Robert B. B. Moorman, Dept. of Civil Engineering, Univ. of Missouri, spoke on "Analysis of Prestressed Concrete Structures." He presented a method whereby the effect of the cable tension can be expressed as a distributed or concentrated load. The analysis, he says, then becomes a simple matter of applying methods with which the designer is already familiar. Dr. Moorman reported afterward that evidently all did not agree with his ideas, as one of his friends reported several shaking their heads in the negative. Be that as it may, they did not discuss the paper from the floor and it is hoped that when the paper is published in the January JOURNAL they will express themselves.

Conservation by design

O. W. Irwin, president, Rail Steel Bar Assn., Chicago, with considerable showmanship presented his paper, "Conservation by Design," in one minute more than he said he would take by dramatically reading one sentence each from the first three or four pages to keep himself within an abbreviated timetable.

His recommendations for conserving steel include: Careful attention to details; deeper concrete sections; elimination of hooks; taking full advantage of new permissible stresses allowed by the revised ACI Code, and especially the adoption of higher allowable tensile stresses for the A305 bars.

He closed his talk by showing a moving picture with Lowell Thomas as narrator, "Rail Steel in the World Today"—"a conservation story which began in 1868."

INFORMAL LUNCHEON

A. Carl Weber, director of research, Laclede Steel Co., and chairman of the local committee, presided at the luncheon. Following a shrill blast of his whistle (evidently he's never without one), he presented a number of "door prizes" donated by an unnamed but well-known manufacturer of steel in the St. Louis area. A. P. Greensfelder, chairman, Fruin-Colnon Contracting Co., drew the cards from a big bowl. How it was arranged will never be known, and Carl Weber still insists it was just chance; but the first five cards drawn bore the names of local committee members and were torn up much to the amusement of the crowd. Mr. Greensfelder even tore up a card himself—evidently his own—before the first prize went to W. R. Wooley of the U. S. Bureau of Public Roads.

The chairman then introduced the officers and directors of ACI, the speakers participating in the technical program, and representatives of the city and educational institutions present.

Dr. Arthur H. Compton, President of Washington University, welcomed the Institute to St. Louis on behalf of the city.



Non-dusting, easy-to-clean "iron-clad" concrete floors that will wear four to six times longer than ordinary concrete floors were obtained in the Grace Street, Richmond, Va., parking garage by the use of Masterplate. It was embedded in the *surface* of the concrete while still in a plastic condition. Masterplate is a combination of specially processed, size-graded iron particles and Master Builders' cement dispersing agent. In addition to the advantages obtained in this parking garage, Masterplate produces non-colored or colored iron-armored floors which are spark resistant, static disseminating, corrosion resistant, non-slip and low cost. Millions of square feet of Masterplate iron-clad concrete floors are in use today and giving excellent service in such structures as — industrial plants, hospitals, warehouses, stores, service stations, ammunition depots, powder plants and similar fields.

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General Chairman A. Carl Weber receives a gift in appreciation of his leadership from William C. E. Becker, Meeting Chairman, on behalf of the committee. President Thomson looks on with interest



Surprise! A shiny new whistle to replace the plastic one Carl uses when making announcements

Building codes

Albert H. Baum, building commissioner, St. Louis, and general chairman, Basic Building Code Committee of the Building Officials Conference of America, gave the luncheon address, *A New Era in Building Codes.*

"Building codes," said Mr. Baum, "come into a category having practically no public, radio, television, or sex appeal. We who have an interest in building codes cannot add lanolin, or irium, or solium to the code to add zest to our product. Codes have no shade-grown flavor or locked-in goodness. They are not even de-bitterized. Codes 'Duzn't do everything' and any other building official *can* make that statement.

"Building codes are most necessary to safeguard our safety, health and welfare," he continued. "Laws and ordinances are necessary because people follow their own whims and fancies until they overstep the bounds of propriety." Modern building codes should be written on a functional or performance basis. They should be flexible and provide for the quick adoption of recent issues of national standards. Building officials should be empowered to approve the use of new materials meeting accepted standards. New codes, he added, should be based upon a nationally recognized model code to secure uniformity in requirements.

Mr. Baum described briefly the work of the Joint Committee on the Unification of Building Codes as an important step in bringing the numerous codes used throughout the country into substantial agreement.

DINNER

Topping the nontechnical activities was the very fine dinner meeting on Tuesday evening. It was estimated that more than 200 members, their wives and guests attended. After an excellent meal, William C. E. Becker, chairman, called on President Thomson to introduce the officers and speakers. Carl Weber again had names drawn for several very nice door prizes. Favors were also distributed to everyone.

Col. F. W. Green, retired president of the St. Louis and Southwestern Railway Co., gave the after-dinner address entitled "Backward or Foreward." He discussed the responsibility of the railroads in maintaining a strong domestic economy; their part in the winning of two world wars; and the problems they face today.

INSPECTION TRIP

The meeting ended with an inspection trip to view outstanding concrete construction under way in St. Louis.

A few blocks from the hotel was the new 500-bed Veterans Administration Hospital now under construction. Total cost of the project is approximately $20,000,000. The main building has structural steel framework encased in concrete with concrete floor slabs.

Next the group visited the new Commission Row construction project, a cooperative venture of wholesale produce merchants. The two main buildings are 114 x 1225 ft and are of precast concrete construction. There is a total of 3.3 miles of wall panel in this job and almost 300,000 sq ft of concrete floors. Precast concrete floor joist construction is used throughout.

After Commission Row, the tour passed the St. Louis Public Housing Development. This project will cost $7,500,000 and is designed to provide living space for 3000 persons. The 13 main buildings have concrete floors, foundations and roofs. The structural engineer is William C. E. Becker of St. Louis and the architects, Hellmuth, Yamasaki & Leinweber, Inc., also of St. Louis. The City of St. Louis has an outstanding record in the promotion of low-cost public housing.

The last stop on the trip (to see Mr. Critzas' tanks, as Carl Weber said) was in South St. Louis to visit Anheuser-Busch Brewery, one of the largest in the world. A detailed inspection was made of part of this plant.

Perlite Institute convention

The Perlite Institute held its 1951 convention in New York City, October 11-12. Not only was it celebrating the Grand Award for distinguished service by trade associations to their industry, but also an enlarged membership of 45.

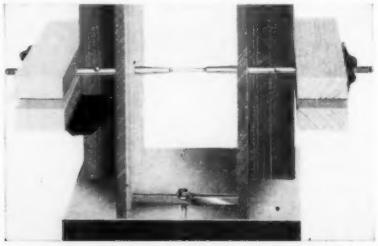
Several fire and sound transmission tests are scheduled for 1952 as well as research at five independent laboratories. Experience on substantial sized operations, such as insulating the roof of the largest cold storage warehouse ever built has confirmed strength, yields and mix proportions issued several months ago.

J. John Brouk is president of the Board of Directors and Wharton Clay is secretary-treasurer. Both are ACI members.

ACPA conducts short course

The American Concrete Pipe Association conducted a short course of instruction for representatives of member companies, in Chicago, Ill., November 26-28, 1951. The school was under the direction of John G. Hendrickson, ACPA research engineer.

Among the many interesting subjects presented and discussed were: hydraulics, manufacturing procedures and methods, loads and bedding, concrete design and laboratory procedures, and problems of sewer construction.



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Preview of Technical Program, 48th Annual Convention

February 26, 27, 28, 1952
Netherland Plaza Hotel
Cincinnati, Ohio

Annual convention program papers were selected by the Technical Activities Committee at its meeting October 29. Eight sessions are proposed on the following subjects:

Prestressed concrete, durability and curing, ultimate load design, materials, design, construction, research, and a panel session on construction problems. Considering that a few last-minute changes might be found necessary, the final program will be about as follows:

TUESDAY, FEBRUARY 26

10:30 a.m., Prestressed concrete

"Relative Economy of Conventional and Prestressed Concrete Reservoirs"
"Field Problems in Constructing Prestressed Concrete Bridge"

2:00 p.m., Durability and curing

"Durability of Railroad Structures"
"Durability of Concrete Pavements"
"Durability of Hydraulic Structures"

A symposium conducted by Committee 612, Recommended Practice for Curing Concrete, which will include:
"Fundamental Principles of Curing"
"Curing Pavements and Canal Linings"
"Curing Structural Concrete"
"Curing Mass Concrete"
"Curing Concrete Products"
"Available Curing Agents and Their Action"

WEDNESDAY, FEBRUARY 27

9:00 a.m., Ultimate load design—Concurrent session

"Why Design by Ultimate Load Theories?"
"Fundamental Relationships in Ultimate Load Design"
"Research on Ultimate Loads"
"Design of Sections by Ultimate Load Theories"
"Load Factors"

9:00 a.m., Materials—Concurrent session

"Thermal Expansion of Aggregates and Concrete Durability"
"Thermal Properties of Concrete and Concrete Aggregates"
"Calcium Chloride in Concrete"
"Creep of Concrete"
"Fly Ash to Reduce Expansion of Certain Cement-Aggregate Combinations"

12:15 p.m., General luncheon

Presentation of Awards
Introduction of New Officers and Directors
Address of the Retiring President

2:30 p.m., Design—Concurrent session

"Designing for Lightweight Structural Concrete"
"Economy of Concrete Beams"
"Developments in Reinforcing Steel and Concepts of Safety"
"Analysis of Skewed Rigid Frames"

2:30 p.m., Construction—Concurrent session

"Applications of Vacuum Concrete"
"Concrete Temperature Control at Pine Flat Dam"
"Small Building Construction"
"Insulation for Protection of New Concrete in Winter"
"Centriline Process of Tunnel Lining"

THURSDAY, FEBRUARY 27

9:00 a.m., Research

The annual open meeting of Committee 115, Research, conducted by S. J. Chamberlin, chairman, and George W. Washa, secretary.

2:00 p.m., Panel session

Past President H. J. Gilkey will serve as chairman of a panel of experts conversant with construction who will be able to answer and discuss questions about concrete on small and medium sized jobs.

Continued on p. 14

Continued from p. 13

Reservations for rooms at the Netherland Plaza Hotel, convention headquarters, should be made by the individual, informing the hotel that the accommodations are desired in connection with the annual convention. Shortly after January 1, all members will receive a preliminary program through the mail—watch for it!

Frank T. Sheets

Frank T. Sheets, president of the Portland Cement Association since 1937, died suddenly on November 3. Mr. Sheets joined the association in 1933 as consulting engineer with a background of 12 years as chief highway engineer and superintendent of highways for the state of Illinois.



For four years prior to his election as president he served as consulting engineer and director of development for PCA. Under his leadership as director of development, many important technological advancements were made in portland cement and concrete. Outstanding among these were the development of soil-cement for paving light traffic roads and streets, and the tilt-up method of construction.

Mr. Sheets was instrumental in bringing about the long-time study of cement performance in concrete to observe the behavior of portland cement in concrete under many varying conditions of climate, soil and water.

During his tenure as president of PCA, Mr. Sheets had an important part in the adoption of highway planning on a national scale and the development of the highway program of the Automotive Safety Foundation.

He served as president of AASHO, the Mississippi Valley Highway Conference, the Central Illinois section of ASCE, and was an official delegate from the United States government to the Pan-American Highway Congress at Rio de Janeiro in 1929.

Mr. Sheets had been associated with ACI since 1938 and was the author of "A Challenge —Shorten the Lag Between Research and Practice," which appeared in the May-June 1938 ACI JOURNAL.

Meyer Hirschthal

Meyer Hirschthal, ACI member since 1934 and member of ACI's Board of Direction 1940-1941, died recently in New York City.

Mr. Hirschthal received his BS at the College of the City of New York in 1899 and CE degree from Columbia University in 1902. He was associated with Ernest C. Ransome, Roebling Construction Co., and Henry Steers, Inc., before joining the engineering staff of the Delaware, Lackawanna and Western Railroad in 1907. During his 44 years with the D. L. & W., Mr. Hirschthal adapted the design of the girderless flat slab to viaducts subject to concentrated loads of railroad locomotives; adapted two-way slab design to concrete approaches of viaducts and devised their truckload distribution. He also applied this design to individual bridges.

After retiring in December 1949, he entered consulting engineering practice and a few months ago organized the firm of M. Hirschthal and Philip King, consulting engineers, New York.

University of Illinois short course

The University of Illinois Small Homes Council will conduct a short course in residential construction at Urbana, Ill., January 16 and 17, 1952. Results of a year-long study of the design and construction of closet-walls will be presented as one of the highlights. The closet-walls, interior partitions which include provisions for storage, were engineered to take advantage of savings possible through the use of trusses, resulting in clear-span roof construction.

Other housing research at the University of Illinois to be discussed includes a report on the effect of frost on foundations of concrete floor slabs, and basementless house construction.

Information regarding registration and fees may be obtained from Robert K. Newton, supervisor of engineering extension, 713½ S. Wright St., Champaign, Ill.

Who's Who This Month

T. Y. Lin

"Lateral Force Distribution in a Concrete Building Story," p. 281, introduces T. Y. Lin, associate professor of civil engineering, University of California, Berkeley.

After graduating from Chiaotung University, China, Professor Lin received his MS degree in Civil Engineering at the University of California in 1933. Returning to China, he worked with the Chinese National Government Railways, becoming chief of design and planning for the Yunnan-Burma Railway. In 1946 he joined the faculty of the University of California, where he now also serves as associate engineer, Institute of Traffic and Transportation Engineering.

An ACI member since 1950, Professor Lin is a member of ACI Committee 314, Rigid Frame Bridges, and chairman of the ASCE Committee on Bridge Loadings.

E. W. Scripture, Jr., S. W. Benedict and F. J. Litwinowicz

"Air Entrainment and Resistance to Freezing and Thawing," p. 297, is the joint contribution of E. W. Scripture, Jr., S. W. Benedict and F. J. Litwinowicz, The Master Builders Co., Cleveland, Ohio. For biographical information on these authors, see pages 7 and 8 of the November 1951 ACI News Letter.

Peter J. Doanides

Peter J. Doanides, chief engineer, Concrete Development Corp., Ltd., and Vacuum-Concrete (Africa) (Pty) Ltd., Johannesburg, Union of South Africa, describes "Streamlined Vacuum Concrete Buntons for Mine Shafts," p. 309.

Born in Greece, Mr. Doanides graduated from the Athens University of Engineering Science, where he was assistant professor for 11 years. Prior to World War II, his activities included design of earthquake resistant structures for the towns of Corinth and Loutraki, Greece. He was also responsible for research work on the use of Isteg steel and its subsequent introduction and application in Greece.

During World War II, Mr. Doanides was a lieutenant with the Royal Hellenic Navy and later was sent on a technical mission with the Royal Hellenic Embassy in Washington, D. C. Following World War II, he became interested in precast concrete especially in the vacuum concrete process and was responsible for its introduction in South Africa.

Mr. Doanides, ACI member since 1944, is also affiliated with ASCE, ASTM and the Society of American Military Engineers.

Felix Candela

Felix Candela, architect, Mexico City, is the author of "Some Simple Types of Concrete Shells," p. 321.

In 1935 he received a degree in architecture from the Escuela Superior de Arquitectura de Madrid, Spain. His interest in structural problems began early in his career, being given a fellowship in 1935 by the Academia de Bellas Artes de San Fernando, Spain, to study new trends in reinforced concrete design in Germany.

He has been in general practice as an architect in Mexico City and recently founded "Cubiertas 'ALA,' S.A.," a firm specialized in shell structures, and "Aeroblock, S. de R.L.," which will produce foamed concrete.

He is the author of papers on reinforced concrete design and shell structures in Spanish and Mexican professional periodicals.

Mr. Candela has been an ACI member since 1947 and is associated with the Institut Technique du Batiment et des Travaux Publics and the International Association for Bridge and Structural Engineering.

Paul Jakowlew-Herbaczewski

"Concrete Footings for Walls and Columns," by Paul Jakowlew-Herbaczewski, senior bridge designer, Thomas Worcester, Inc., Boston, Mass., appears on p. 333.

His experience includes 35 years of design and construction of railways, bridges, steel and reinforced concrete buildings.

For ten years, he worked as a bridge designing and railway construction engineer

Continued on p. 19

Positions and Projects — ACI Members

N. T. F. Stadtfeld

N. T. F. Stadtfeld, Board of Water Supply, New York, N. Y., has been appointed as an additional ASTM representative on ASA Sectional Committee A-21 on Cast Iron Pipe.

Burggraf named director

Fred Burggraf was recently appointed director of the Highway Research Board, Washington, D. C. He has been connected with research relating to highways since 1919 and his career includes engagements with the National Bureau of Standards, Illinois Division of Highways and Calcium Chloride Assn. He served as research engineer of the HRB from 1928 to 1932 and returned as assistant director in 1940. He was named associate director in 1945.

ASCE elects Ohrt to honorary membership

Frederick Ohrt, manager and chief engineer, Board of Water Supply of the City and County of Honolulu since 1929, has been elected to honorary membership in ASCE. The honorary membership award was made on October 24, 1951, during the ASCE annual convention in New York City.

Mr. Ohrt has initiated broad-scale research work and has been responsible for the engineering of extensive ground-water development construction.

Haley leaves ACPA

William A. Haley, III, has resigned as assistant to Howard F. Peckworth, managing director of the American Concrete Pipe Assn., Chicago, Ill., to accept a position on the staff of the Southern Block and Pipe Co., Norfolk, Va.

Lutes elected association director

Harold Lutes, general manager, Layrite Concrete Products Co., Spokane, Wash., and retiring president of the Concrete Products Assn. of Washington, has been elected a director of the association for the coming year.

Jakobsen retires

Bernhard F. Jakobsen retired recently after 20 years of service as a civil engineer with the Corps of Engineers, South Pacific Division, Oakland Army Base, Calif. Mr. Jakobsen served as a design engineer and expert consultant on many dam and power projects. Twice awarded the Norman Medal of ASCE, in 1948 he was a member of the United States delegation to the International Conference on Soil Mechanics and Foundation Engineering.

Bromilow appointed department head

Frank Bromilow, associate professor of civil engineering at the University of Florida since 1948, has accepted appointment to the position of professor and head of the department of civil engineering at New Mexico A. & M. College. Previously he was chief engineer for the Plasteel Corp., Washington, D. C.

Price appointed USBR laboratory head

Walter H. Price recently was appointed head of the U. S. Bureau of Reclamation's engineering laboratories, Denver, Colo.

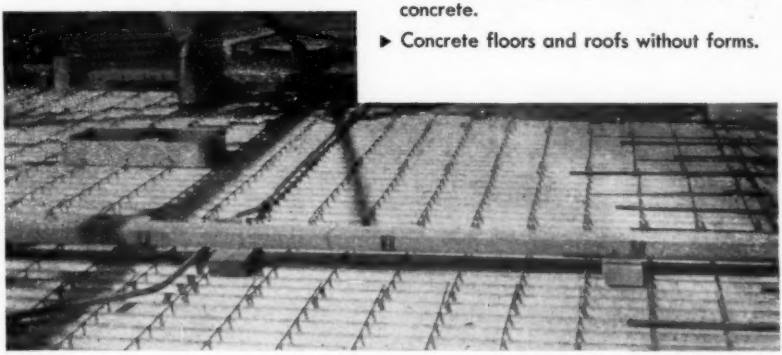
A 1930 civil engineering graduate from Tulane University, Mr. Price has been with the Bureau of Reclamation since that year and a staff member of the Bureau's Denver laboratories since 1934. He has had wide experience in hydraulic studies and experiments, structural design and research, and materials testing and research. Immediately prior to his appointment as head of the engineering laboratories, Mr. Price was head of the materials laboratories.

He is a member of ASCE, Colorado Society of Engineers and Sigma Xi. A member of ACI since 1937, he is chairman of Committee 210, Resistance to Erosion in Hydraulic Structures; Committee 613, Recommended Practice for Proportioning Concrete Mixes; and is a member of ACI's Board of Direction.

Mr. Price was appointed following the resignation of Robert F. Blanks, who accepted the position of vice president of Great Western Aggregates, Inc., Denver, Colo.

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- Concrete floors and roofs without forms.



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COFAR sheets hot-dip galvanized, with specification heavy galvanizing, provide complete permanence for normal interior exposures. Concrete and steel permanently bonded by chemical union of calcium zincate for all normal loads. Fire resistance for any exposure by lightweight, modern ceiling plaster or fibre protection.

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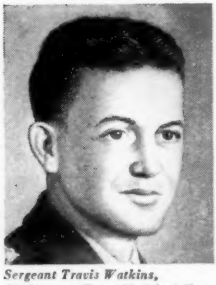
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Medal of Honor



Sergeant Travis Watkins, Gladewater, Tex.—Medal of Honor



Lieutenant Frederick Henry, Clinton, Okla.—Medal of Honor



Sergeant Charles Turner, Boston, Mass.—Medal of Honor

This is the season when you think of stars. The one over Bethlehem. The ones on Christmas trees.

But this year remember another star, too—the one on the Medal of Honor. And make a place in your heart for the brave, good men who've won it. Men who, oftener than not, made the final, greatest sacrifice—so that the stars on your Christmas tree, and the stars in your country's flag, might forever shine undimmed.

Right now—today—is the time to do something important for these men who died for you. You can, by helping to defend the country they defended so far "above and beyond the call of duty."

One of the best ways you can make defense *your* job, too, is to buy more . . . and more . . . and more United States Defense Bonds. For your bonds help strengthen America. And if you make this nation strong enough you'll create, and keep, the peace for which men died.

Buy Defense Bonds through the Payroll Savings Plan where you work or the Bond-A-Month Plan where you bank. Start today!

Peace is for the strong...Buy U. S. Defense Bonds

Gould reports on European structural engineering

At the annual convention of the Structural Engineers Association of California, John J. Gould, consulting engineer, San Francisco, Calif., and member of ACI reported on his recent trip to Europe in a paper entitled "Observations on Structural Engineering in Europe."

Mr. Gould said that for 30 years a great amount of research has been devoted to precast and prestressed concrete. Recently, processes of preheating, preshaking and other new construction schemes have made prestressed concrete not only safe but also economical.

In London, England, Mr. Gould observed the construction of a 10-story office building of reinforced concrete frame and floor construction with exterior brick walls. In view of the shortage of lumber, the entire building was built with a minimum of wood forms.

In Germany, precasting of concrete is generally confined to mass articles in factories. Small precast and prestressed joists are being manufactured in forms roughly 100 ft long. In the forms are placed high strength wires which are put under initial tension by means of hydraulic jacks placed at either end. After wires are stressed, a very dry concrete is then placed in the forms. Steam pipes are located on either side of the forms so that immediately after the concrete is placed, the setting of the concrete is accelerated by artificial heat. After the concrete has set, the joists are then prestressed by transferring tension in the wires into the concrete simply by releasing the hydraulic jacks. This compresses the joists by several inches. In some cases, high-early cement is being used, so that generally speaking, concrete joists are removed from the forms 24 hours after casting.

Mr. Gould discussed various methods of prestressing concrete with prominent European designers. He stated that Dr. P. W. Abeles has directed a large number of tests on prestressed concrete and, in their discussion, Dr. Abeles asserted that tests have proved that fatigue does not affect well-bonded wires in prestressed concrete, whether these wires are tensioned or untensioned, while nonbonded wires fail at approximately half the stress under fatigue.

In prestressing concrete bridges by the Baur-Leonhardt method, Mr. Gould observed that a number of small wires are put together in a steel box in the concrete forms of the girder to be stressed. In plan, the wires are looped around a gigantic concrete block with hydraulic jacks placed between the blocks and main girders. After the concrete has been set for about two days, the concrete is prestressed slightly so as to take up first initial shrinkage. After about 10 days, the same procedure is followed and at the end of 28 days, the final prestressing is then accomplished by applying desired tension in the cables.

In concluding his talk, Mr. Gould declared that, in visiting cities damaged by war, he was impressed with the much greater damage caused by fires rather than bombing, and that industrial structures which had been built of fireproofed steel or reinforced concrete generally suffered little damage.

Ryan forms organization

Alfred J. Ryan, Denver, Colo., consultant, announces the formation of an organization to be known as A. J. Ryan and Associates, Engineers. The firm will provide complete services in the fields of civil, electrical, mechanical and structural engineering.

Breitfuss joins ACPA staff

Thomas K. Breitfuss has joined the American Concrete Pipe Association, Chicago, Ill., as assistant to Howard F. Peckworth, managing director. Mr. Breitfuss is a member of Tau Beta Pi, ASTM, ATAE, and is Junior Treasurer of the Illinois Section of ASCE.

Who's Who This Month

Continued from p. 15

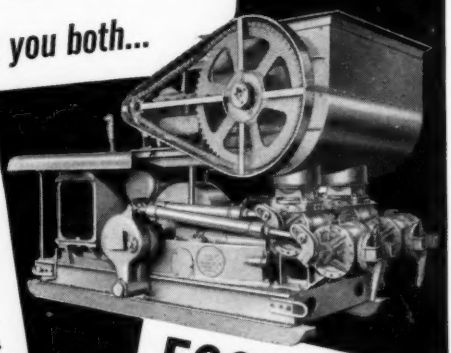
in Russia and for two years was employed by Bethlehem Steel Co., Petrograd, Russia.

From 1925 - 1928, Mr. Jakowlew-Herbaczewski was chief structural engineer with Ulen Company, American contractors for reinforced concrete, in Poland.

He has written more than 30 articles on structural engineering which were published in Polish engineering periodicals and for many years was a member of Polish committees of standards for steel and concrete.

In 1938, he was decorated with the Golden Cross of Merit for services rendered to Polish State Railways.

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CONSTRUCTION MACHINERY

International Association for Bridge and Structural Engineering

Preparations are being made for the 4th Congress of the International Association for Bridge and Structural Engineering to be held at Cambridge University, England, August 25 to September 5, 1952. Technical sessions will be held during the first week of the Congress while the second week will be devoted to a government reception and dinner, tours and inspection trips.

The International Association for Bridge and Structural Engineering is a central international clearinghouse of information and a sounding board for discussion of new developments, research and methods in structural and construction engineering. Its aim is to promote collaboration between engineers of different countries for the exchange of ideas, of theoretical and practical knowledge, and of the results of research in these fields. Members are informed by reports and printed publications of the results of practical experiments and research. In addition, congresses are organized at four-year intervals, to advance and expand the task by personal contact with members.

Although the association is managed by a permanent committee on which each country has representation in proportion to its membership, the organization functions on a national basis. A U. S. Council, elected by the membership, formulates policies and administers the affairs of the association here. Leo Coff, J. M. Garrelts, T. C. Kavanagh, F. W. Panhorst, John I. Parcel, R. H. Sherlock, D. B. Steinman and George Winter, comprise the present council. All are members of ACI.

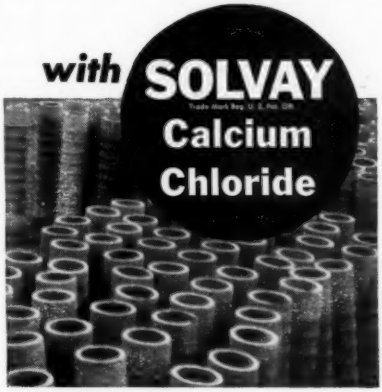
Road builders plan 50th meeting

American Road Builders Assn. will hold its 50th Anniversary Meeting in Houston, Texas, January 21-24, 1952. Because of its significance in the history of ARBA, an especially comprehensive and interesting program is being arranged for the meeting.

Errata

Definition of f_t under Eq. 4, "Load Carrying Capacity of Dowels at Transverse Pavement Joints," p. 181, October 1951, JOURNAL should read $f_t = 10\sqrt{f'_c}$.

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JC-12

New Members

48 Individual, 2 Corporation, 10 Junior and 29 Students make a total of 89 member applicants for October to bring the membership to an all-time high of 5441.

That 21 are from outside the United States shows growing interest in ACI in other parts of the world.

Individual

AGBAYANI, LUCAS F., Quezon City, Philippines (Struct. Design, Dept. of Engrg., Quezon City)

ALEFANDAKIS, YANI, Detroit, Mich. (Struct. Engr., Giffels & Vallet, Inc.)

ANTENEN, JAY FREDERICK, Hamilton, Ohio (Gen. Supt., Antenen Engrg. Co., Inc.)

BALEDON B., ARTURO, Mexico, Mexico (C. E., Ingenieros Civiles Asociados)

BLUCHER, CONRAD M., Corpus Christi, Texas (Pres., Blucher & Naismith, Inc.)

BORCHELT, C. T., Peoria, Ill. (Cons. C. E.)

BOTCH, PHILIP M., Bellevue, Wash. (Struct. Engr., John Graham & Co.)

BRICMONT, ARTHUR E., JR., Oklahoma City, Okla. (Concrete Engr., The Dolese Co.)

CEBULA, RICHARD S., Olympia, Wash. (Dir. of Engrg. Dept., Instr. in Struct. Engrg., St. Martin's College)

CHACKO, K.C., Notre Dame, Ind. (Lecturer in Charge, Univ. of Travancore, India)

COMSTOCK, PHILLIP D., Louisville, Ky. (Gen. Supt., Engr., Ohio River Sand Co., Inc.)

FLOYD, ROBERT W., San Diego, Calif. (Concrete Engr., San Diego Transit Mixed Concrete Co.)

FURSH, STEFAN J., San Carlos, Calif. (Assoc. Constr. Super., Calif. Dept. of Public Works, Div. of Arch.)

GIEROW, ALBERT O., Jackson, Mich. (Struct. Design, Commonwealth Assocs., Inc.)

GOODLIN, MILTON A., Honolulu, T. H. (C. E., Design, Civil Aeronautics Admin.)

HOEJGAARD, K., Copenhagen N, Denmark (Hoejgaard, & Schultz A/S)

HOYTE, LENNOX P., Trinidad, B. W. I.

HUGHES, HARRY D., Seattle, Wash. (Gen. Constr. Supt., Cmdr. Zechella)

HUTCHINGS, E. T., Louisville, Ky. (Arch., Engr.)

JOHNSON, J. H., Chicago, Ill. (Johnson & Johnson)

KAUCHER, WALTER, Los Angeles, Calif. (Gen. Contr.)

KEEN, L. B., Providence, R. I. (L. B. Keen Co.)

LORANCE, BERNARD W., Eldorado Springs, Colo. (Chf. Struct. Engr., I. H. Buell & Co.)

MADAN, G. S., Calcutta, India (Chf. Engr., The Commissioners for the Port of Calcutta)

MAHADEVA, V. N., Fort Bombay, India (Cons. Engr.)

McDOWELL, C. T. J., Narborough near Leicester, England (Empire Stone Co., Ltd.)

NAGABEER, D. B., Calcutta, India (R. C. Design & Super., Concrete Assn. of India)

NASH, DONALD E., Welland, Canada (Super. Engr., Vice-Pres., G. E. Nash, Ltd.)

PEDERSEN, K. I., Caracas, Venezuela (Constr. Super., Empresas Campenon-Bernard de Venezuela)

PETERSON, C. V., Victoria, Texas (Jr. Engr., DuPont Co.)

POPKIN, A. G., San Francisco, Calif. (Gen. Mgr., Consumers Rock & Cement Co.)

RODWAY, L. F., Toronto, Canada (Constr., Struct. Engr., Canada Cement Co.)

ROSE, A. H., Snyder, Texas (Supt. of Constr., R. F. Ball Constr. Co.)

RUNYON, N. E., Jacksonville, Fla. (Chf. Engr., Cosco Products Co.)

SCHLUCKEBIER, PAUL D., Jackson, Mich. (Design, Drafting, Commonwealth Assoc., Inc.)

SHERIDAN, JOSEPH H., Manchester, Tenn. (Field Insp., Rust Engrg. Co.)

SHROPSHIRE, GEORGE DAMON, JR., Roanoke, Va. (Asst. C. E., Hayes, Seay, Mattern & Mattern)

SILER, WILLIAM C., Lockland, Ohio (Design, Insp., Jackson & Moreland)

SINGER, ALEXANDER, Mt. Vernon, N. Y. (Cuzzi Bros. & Singer, Inc.)

SMITH, GERALD M., Manhattan, Kans. (Asst. Prof., Kansas State College)

STANCER, WILLIAM A., Tacoma, Wash. (County Road Engr., Board of County Commissioners, Pierce County)

STENDELL, ELVIN C., San Francisco, Calif. (Gen. Contr.)

STERN, LYLES A., Dallas, Texas (Sales Engr., Pittsburgh Testing Lab. of Texas)

STEWART, JOHN JOSEPH, Wilmerding, Pa. (C. E., Grad St., Carnegie Tech.)

TURNER, J., Worcester Park, Surrey, England (Engr., Civil Engrg. Research Labs.)

VANAGS, ALFREDS R., Melbourne, Australia (C. E., Design, Commonwealth of Australia, Dept. of Works & Housing)

WILBORN, JACK K., Marietta, Ga. (Chf. Struct. Engr., Stevens & Wilkinson)

ZAWADZKI, BENON JAKUB, London, England (Designer, Twisteel Reinforcement Ltd.)

Corporation

PUBLIC WORKS CENTER, San Francisco, Calif. (E. V. Earle)

WILLCOX CONSTRUCTION CO., INC., Long Island, N. Y. (Henry Willcox)

Junior

ACKERMAN, STEVEN, Brooklyn, N. Y. (Struct. Engr., Brown & Blauvelt)

CONESA, ANTULIO, Ponce, P. R. (Drafting, Designing, Constr. Insp., H. P. Conesa Assoc.)

GRANT, ROBERT ULYSSES, Laramie, Wyo.

GARCIA GRUBER, ROBERTO, Caracas, Venezuela (Ingeniero al Servicio de la Direccion de Obras de Riego del Ministerio de Obras Publicas)

HSU, SERGE J. C., Estado do Rio, Brazil (C. E., Rio de Janeiro Tramway Light & Power Co.)

LUCAS, WALTER M., Chicago, Ill. (Drafting, Designing, Engineering Service Bureau)

PAYANT, DONALD O., Kohler, Wis. (C. E., Kohler of Kohler)

PRITCHARD, BRIAN P., Glamorgan, S. Wales, Great Britain (PCA)

TOMLINSON, JACK MURRAY, Hamilton, Canada (Design, City Engrs. Dept.)

TURSKI, ALVIN RAYMOND, Urbana, Ill. (Struct. Research, Univ. of Ill.)

Student

ANSON, HAROLD, New York, N. Y. (Columbia Univ.)

BAYER, JOHN A., Urbana, Ill. (Univ. of Ill.)

BERRY, SYDNEY B., (Miss) Champaign, Ill. (Univ. of Ill.)

CARRIZALES ARREAZA, LUIS ALBERTO, Caracas, Venezuela (Ministry of Public Works)

D'AGOSTINO, THOMAS S., Pittsburgh, Pa. (Carnegie Inst. of Tech.)

DIXON, PAUL E., Urbana, Ill. (Univ. of Ill.)

DOCHERTY, WILLIAM T., Champaign, Ill. (Univ. of Ill.)

EL-KHURI, SAMIR, Bethlehem, Pa. (Lehigh Univ.)

FIELD, ROBERT D., Champaign, Ill. (Univ. of Ill.)

FOTE, WILLIAM D., JR., Notre Dame, Ind. (Univ. of Notre Dame)

GRAHAM, HARRY J., Columbia, Mo. (Univ. of Mo.)

GULDE, JOSEPH F., Notre Dame, Ind. (Univ. of Notre Dame)

HOELTERHOFF, FRANK W., Chicago, Ill. (Univ. of Ill.)

HOLTON, DAVID S., Corona, Calif. (F. Thomas Collins)

JOMBOCK, JOHN R., Virden, Ill. (Univ. of Ill.)

LINSENMEYER, CHARLES MICHAEL, Notre Dame, Ind. (Univ. of Notre Dame)

LOCKE, JOHN P., Champaign, Ill. (Univ. of Ill.)

MAYO, RICHARD W., Bethlehem, Pa. (Lehigh Univ.)

McCLAIN, CLARENCE B., Urbana, Ill. (Univ. of Ill.)

NEYER, RAYMOND, South Bend, Ind. (Univ. of Notre Dame)

OKOGA, LUTHER T., Aiea, Oahu, T. H. (Univ. of Hawaii)

Continued on p. 25

Use the New Numbering System When You Specify Bars



Bethlehem Reinforcing Bars are now identified by numbers from 2 to 11, in accordance with Simplified Practice Recommendation R 26-50 of the U. S. Dept. of Commerce. Each number indicates approximately the eighths-of-an-inch in the nominal diameter of the bar.

Bars #9, #10 and #11 are equal in weight and nominal area to the old type 1 in., 1⅛ in. and 1¼ in. square bars. Except in the case of the #2 plain round bar, the numbers are branded at regular intervals on each deformed bar.

Bethlehem Bars are rolled as round sections. Because no change has been made in either the standard unit weights or nominal cross-sectional areas, you do not need to alter any design practice.

Only new-billet steel meeting ASTM Specification A15 goes into the improved Bethlehem Reinforcing Bar. The bar provides a firm anchorage in concrete because its high, closely-spaced lugs meet the deformation requirements of ASTM Specification A305. These alternately-sloped lugs minimize slips at working loads and help prevent wide tensile cracks in the concrete.

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International Congress on Prestressed Concrete

By P. G. BOWIE*

The International Congress on Prestressed Concrete held at Ghent, Belgium, September 10-13, 1951, marked the 75th anniversary of the Association of Engineers of the special schools of the University of Ghent (AIG) and was held jointly with a Congress on Science as applied in the textile industry.

Included in the Congress were the presentation and discussion of 53 papers; official receptions and dinners; acceptance by Eugene Freyssinet of honorary membership in AIG; and visits to factories and works under construction.

The papers presented were mainly descriptions of prestressed work carried out by the authors with particular reference to continuity and portal frames. Several topics of special interest were the fire resistance test of a 38-ft prestressed beam up to 1300 C; the Philadelphia bridge; ultimate strength in bending; the prestressing of natural stone; small-span sectional bridges; and continuous beams.

The inspection trip included the bridge at Hautrage of about 150 ft span and 80 ft width, built of box cells 3 ft long of only three or four different patterns, and the Franki-Smet bridge with its special anchorages and cable casing for in-situ construction. Both bridges were examples of the continued development and improvement going on in this part of Europe.

The organization of the entire Congress reflected great credit on Professor Magnel and other members of the University staff associated with him.

*Chief Structural Engineer, Cement & Concrete Assn., London, England.

Honor Roll

February 1—October 31, 1951

Only 38 of the total membership—5441—are top member proposers having two or more credits for the current year. Many more should be listed as ACI Member-getters before February 1.

The more active members ACI has, the more varied ideas shared. The benefits of ACI are desired by many requiring the opportunity to join that only members can give.

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Newlin D. Morgan, Jr. (Mich.)	17½
Eddy Hernandez (La.)	10½
James A. McCarthy (Ind.)	10
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Oscar A. Nunez G. (Venezuela)	7½
L. G. Farrant (Fla.)	6
H. J. Gilkey (Ia.)	6
Samuel Hobbs (Calif.)	6
Alvin C. Loewer, Jr. (Pa.)	5
Jose A. Vila (Cuba)	5
F. Thomas Collins (Calif.)	4
Oliver J. Julian (Mass.)	4
Frank Kerekes (Ia.)	4
Alexander Klein (Calif.)	4
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Fernan Rodriguez-Gil (Venezuela)	3½
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E. L. Howard (Calif.)	2½
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F. T. Mavis (Pa.)	2½
C. H. Scholer (Kans.)	2½
Jacob Adler (Israel)	2
Donald Agrimson (S. D.)	2
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Robert J. Deitrich (Md.)	2
Frank L. Ehasz (N. Y.)	2
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New Members

Continued from p. 22

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SANTAELLA, EDUARDO JOSE, Pittsburgh, Pa. (Carnegie Inst. of Tech.)
STEIN SUED, RODOLFO, Havana, Cuba (Univ. of Havana)
SZUBA, JOHN P., Chicago, Ill. (Univ. of Ill.)
YILDIRIM, IRFAN, Champaign, Ill. (Univ. of Ill.)
YUNGCK, ROBERT JOSEPH, Alton, Ill. (Univ. of Ill.)

Tools, Materials, Services

Under this heading note will be made from time to time of producer literature of presumed technical interest (and available from its source for the asking) to ACI users of tools, equipment, materials, accessories and special services.

High-speed paver

A new dual drum paver, designed with ultra high-speed operating cycles, is being manufactured by the Worthington Pump and Machinery Corporation to meet modern paving conditions. Typical of the speed claimed for the new Worthington-Ransome 34E Dual Drum Paver (Model WP) is a boom bucket travel rate of 256 ft per minute, the power loader skip requiring only 6½ seconds to travel from the ground to a discharge position of 56° low slope. Transfer from the first to the second mixing compartment also requires about 6½ seconds, depending upon consistency of the concrete. Discharge into the boom bucket is another 6½ second operation. Boom swing is a full 171°.

The new paver retains the exclusive hydraulic bucket, permitting complete control of rate of concrete discharge under all conditions. Increased breaking area, more efficient boom raising and lowering machinery, a redesigned drum, and entirely new crawlers and frames are other features of this new machine.—Construction Equipment Div., Worthington Pump and Machinery Corp., Dunellen, N. J.

Vermiculite publication revised

The Vermiculite Plaster Fireproofing booklet has been revised recently. A summary of floor and ceiling fire tests lists the fire ratings obtained by various floor constructions protected with a vermiculite plaster ceiling and the fire ratings obtained by this ceiling when attached, furred and suspended from primary members. Typical details for fireproofing primary and secondary members with a ceiling of vermiculite plaster are illustrated. Included in the revised booklet are typical column fireproofing details, including gypsum lath and vermiculite plaster protections, as well as a summarization of the fire test results on vermiculite plaster column protections applied to both metal lath and gypsum lath, and the ratings for vermiculite plaster solid partitions employing metal lath and gypsum lath.—Vermiculite Institute, 208 S. LaSalle St., Chicago 4, Ill.

Double pitch roller chains

Baldwin-Rex double pitch roller chains are pictured and described in detail in a new bulletin released by Chain Belt Co. of Milwaukee.

Continued on p. 26

ALPHABETICAL LIST OF ADVERTISERS

(Page numbers refer to News Letter)

Baldwin-Lima-Hamilton Corporation 8
Bethlehem Steel Company 23
Chain Belt Company 20
Granco Steel Products Company 17
Intrusion-Prepakt, Inc. 2
Lone Star Cement Corporation ..... Inside front cover
Master Builders Company 10
Solvay Sales Division, Allied Chemical and Dye Corporation 21
Charles R. Watts & Company 25
Williams Form Engineering Corporation 12

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Tools, Materials, Services

Continued from p. 25

The bulletin describes how these chains can often effect both cost and weight reductions in drives and conveyors. Charts, photos, diagrams, and tables show in detail all Baldwin-Rex double pitch roller chains—both drive and conveyor types—and the standard attachments available. There is also a description of double pitch chain sprockets.—Chain Belt Company, 1600 W. Bruce St., Dept. PR, Milwaukee 4, Wis.

World's largest concrete mixer

A concrete mixer capable of mixing 168 cu ft of concrete in one batch is now helping build the $400,000,000 U. S. Steel Corporation's new plant at Morrisville, Pennsylvania.

The largest drum-type mixer ever manufactured, it was made for the Warner Corporation of Philadelphia, Pa., by the Worthington Pump and Machinery Corporation. It is capable of filling a 6¼ cu yd Hi-up truck-mounted agitator to capacity with a single batch.—Worthington Pump and Machinery Corp., Harrison, N. J.

Coralux—Annealed perlite aggregate

An 8-page booklet containing helpful information and specifications on the application of "Coralux" acoustical plaster, plaster aggregates and insulating concrete aggregate has been prepared by F. E. Schundler & Company, Inc., as a guide for architects, heating engineers, contractors, builders and dealers.—F. E. Schundler & Co., Inc., 504 Railroad St., Joliet, Ill.

Steele retires

Byram W. Steele has retired after nearly 31 years of Federal service on dam design and construction—the first 16 years with the Bureau of Reclamation and nearly 15 years with the Corps of Engineers. He was in charge of storage dam design and materials investigations for the Bureau and his work with the Corps of Engineers was along the same line.

Thank you

ACI headquarters staff wishes to thank members of the Board of Direction and others who replied to the inquiry concerning former Institute officers and directors so that as complete a list as possible may be published in the next ACI Directory. The response was admirable; the time and effort spent by many of ACI's Members is sincerely appreciated.

New cement plants in Jerusalem

Cement production is to be almost doubled by the end of next year, when two new plants now being constructed in the Jerusalem corridor are expected to start operating at a combined annual capacity exceeding 300,000 tons. One of the plants is being constructed by the Shimshon Co. near Hartuv and the other by the Nesher Co. outside Ramle.

Title No. 48-24

Building Multistory Reinforced Concrete Tanks*

By E. J. CRITZAS†

SYNOPSIS

A brief description of some of the field problems encountered in building rectangular multistory reinforced concrete tanks used for the storage or fermentation of beer. Tank drainage and plant ventilation required special formwork to provide the necessary warped or sloped surfaces. Specified coverage and placement of reinforcement led to development and use of special precast concrete spacer blocks and metal chairs. Varying job conditions dictated use of both metal and plywood forms and rough surfaces required for bonding odor- and taste-free lining imposed special concrete finishing techniques.

INTRODUCTION

Two types of tanks were built; the first designed to carry liquid load only, while the second was designed to withstand an internal gas pressure of 15 psi in addition to the liquid load. Both types were rectangular, with width varying from 12 to 15 ft, height of 7 ft 6 in. to 9 ft, and length of 40 to 60 ft. The tanks were arranged in groups or blocks of 2 or 3 per floor (Fig. 1), with common intermediate walls and cantilever walkways attached to some of the exterior walls. They were built from three to six stories high, and were designed as multistory rectangular frames, with the ceiling of the lower tier forming the floor slab of the upper tank (Fig. 2).

Two, four, or six such blocks were incorporated in one building with a service corridor approximately 10 ft wide between blocks. The entire tank structure was enclosed by insulated brick and concrete parapet walls and an air space was provided between the tank blocks and the exterior walls (Fig. 3). The building is air conditioned and maintained at 38 to 45 F.

FOUNDATIONS

The tank structure was supported on the longitudinal wall sides only, with beams spanning between these walls to support the two end walls. In most cases, the supporting walls were 10 to 12 ft high forming basement rooms for housing air-conditioning equipment, pumps and other machinery. In others, where no basement space was necessary, the tanks were supported by longitudinal walls about 4 ft high, thus providing an air space between

*Presented at the ACI St. Louis Regional Meeting, St. Louis, Mo., Oct. 30, 1951. Title No. 48-24 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE: No. 5, Jan. 1952, *Proceedings* V. 48. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than May 1, 1952. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Construction Engineer and Vice-President, Borsari Tank Corp. of America, New York, N. Y.

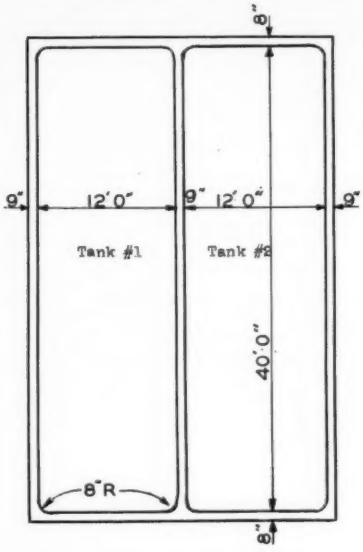


Fig. 1—Typical plan of tank blocks

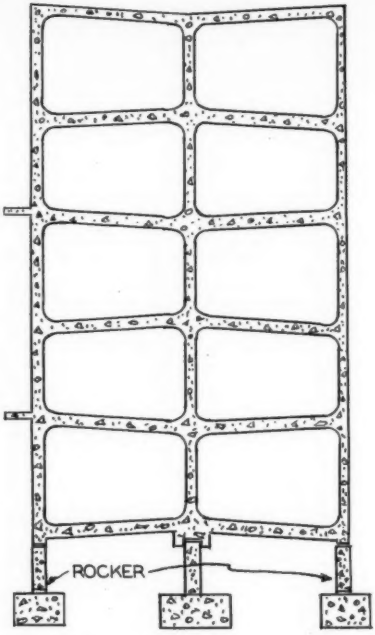


Fig. 2—Typical cross section of a two-tank block five stories high

the ground and the first tank floor slab. Design considerations called for hinged supports at the exterior longitudinal walls with a fixed support for the center wall. Fig. 4 shows the reinforcement for an outside supporting wall.

FORMWORK

Operating conditions called for draining and servicing tanks from the corridor or front side only, and in addition required that all ceilings slope upward toward the corridor side to permit accumulation and removal of gases developed during the manufacturing process. A variable tank floor slab thickness appeared necessary, presenting considerable construction difficulty. However, a constant floor thickness was developed by using a patented method of warping the floor surface instead of using a plane surface. This type of surface is shown in Fig. 5, and is generated by a straight line moving on two nonparallel straight lines. This required careful forming of the walers and joists supporting the floor slabs. Plywood panels, resting on the accurately placed joists, readily conformed to the slightly warped form surface.

All wall intersections or corners inside tanks were beveled or rounded and specifications called for a construction joint between floor and walls 12 to

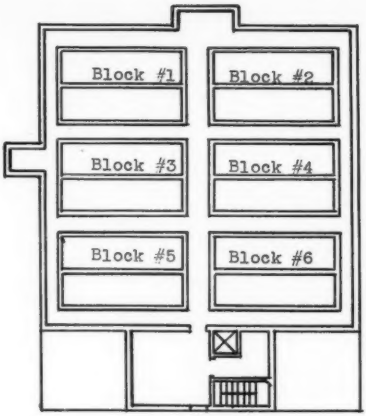


Fig. 3—Typical floor layout of building enclosing six tank blocks



Fig. 4—Reinforcement of rocker supporting wall

18 in. above the finished floor line to eliminate a feather-edge joint at the floor line and avoid a construction joint at the point of maximum moment. This presented the problem of effectively supporting the stub portion of the tank wall so that it could be placed monolithic with the floor slab. Several methods were tried before a final form design providing a curved metal section (Fig. 6), supported by specially designed welded wire chairs (Fig. 7) was developed. The X-type chair was used for intermediate walls, while for exterior walls a similar type was developed, using a locking tie with it. The curved metal sections were standardized in various lengths up to 8 ft, and were bolted together on the job. For the intersection of three walls, a metal corner was provided consisting of $\frac{1}{8}$ of a sphere. To avoid jamming the metal forms during stripping, a wood filler strip varying from 2 to 3 in. thick was

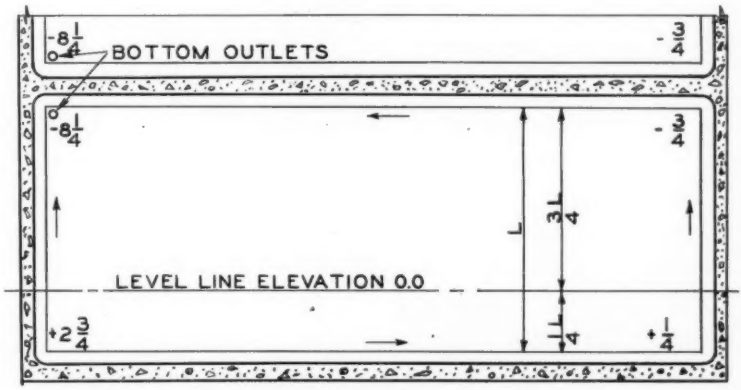


Fig. 5—Tank floor drainage arrangement

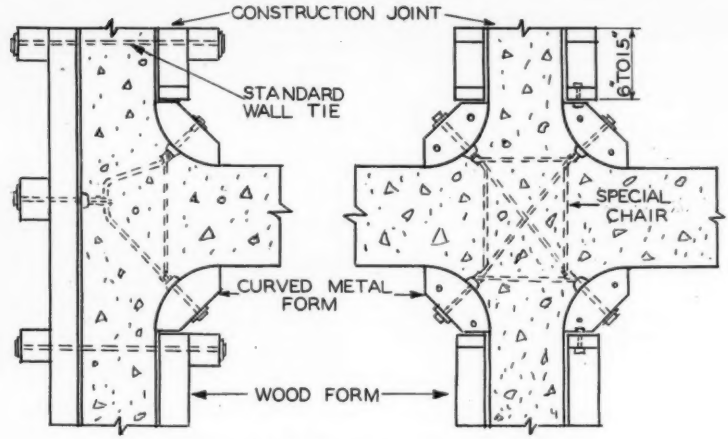


Fig. 6—Curved metal forms for center and outside walls

used for each straight tank side. This filler was of the same cross section as the curved metal form; by making it in two wedge-shaped sections bolted between two metal sections, it was possible to start form stripping at this point without damaging the metal forms. The curved metal sections also provided a satisfactory guide for screeding the concrete floor at the proper elevation; by sliding a wood or metal screed on the straight edge of the metal form the desired warped finished surface was obtained.

The same type of curved metal form was used to form the rounded vertical tank corners.

For the vertical walls, both wood and metal forms were tried, but it was found that metal forms were suitable only where at least 10 re-uses were necessary. Plywood forms were preferred and were designed to rest on 2 x 4 studs bolted on top of the curved metal sections and arranged in panels. By suitable marking of panels and care in stripping, 6 to 8 re-uses, with minor repairs, were possible.

For the pressure tanks, access to the interior was provided by a circular bronze manhole door framé 26 in. in diameter, cast in the concrete walls and properly anchored (Fig. 8). In this case all panel forms had to be made not more than 24 in. wide to permit removal without wrecking the panel. With the nonpressure type tanks, it was possible to provide a larger opening accommodating panels 4 ft wide. The length of the panels was usually the full height of the tank wall. Standard wall ties were used for all walls.

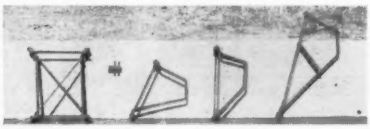


Fig. 7—Special chairs for supporting curved metal forms

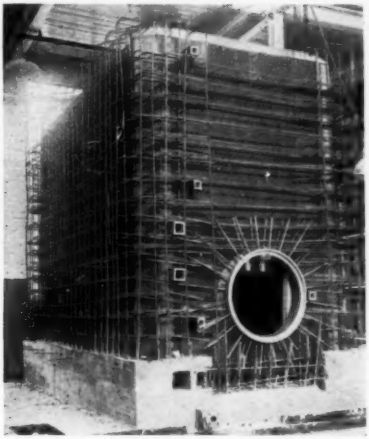


Fig. 8—Bronze manhole door in forms for single tank

SURFACE TREATMENT OF INTERIOR TANK WALLS

The specifications called for roughening all interior surfaces of tank walls to provide good adhesion for the lining material. This was accomplished by two methods. First, by using a liquid chemical painted directly on the plywood forms, or on masonite lining attached to the plywood form. This coating retarded the setting of a thin film of cement on the exposed surfaces of the concrete walls and ceilings. When the forms were stripped, this thin film was removed by wire brushing, thus exposing the concrete aggregates and providing an excellent bonding surface. The second method consisted of sand blasting or Vacu-blasting the exposed surfaces.

The retarding chemical coating could not be used on metal forms and, unfortunately, was affected by water and had to be protected from rain. For this reason, a combination of the two methods was found most economical, with Vacu-blasting supplementing the chemical treatment where rain or other reasons made it impossible to obtain the desired rough surface.

To get a similar rough surface on the floor, the floor was either sprayed with the retarding material, or brushed lightly before the surface had set, usually about 3 to 4 hours after the completion of the concrete work.

CONCRETE

The strength of concrete specified was 3500 psi minimum, with high-early strength cement. The high-early strength cement was specified mainly to allow rapid removal of inside forms, thus permitting re-use and eliminating fabrication of many sets of forms, and also to shorten the period of wet curing, thus permitting construction of the upper floors without continuously wetting the lower sections. Concrete was wet cured during the first 48 hours by allowing water to trickle through canvas hoses placed on top of the raised construction joints. Floors were protected with waterproof paper.

To obtain the desired concrete strength and avoid honeycombing, a concrete with 4 to 4½ in. slump was specified. This was obtained by using a 5½-bag mix with Plastiment added to it. Concrete cylinder tests usually varied from 4500 to 6000 psi. Ready-mix concrete was preferred where available.

The placing of the concrete presented no difficulties. A platform about 3 ft above finish floor grade of the tank rested on the sides of the curved metal sections, and permitted placing the concrete directly in tank walls and floor from buggies. With wall thicknesses varying from 6 to 9 in. (for nonpressure tanks) and wall reinforcing on both faces, it was necessary to use "C" or ¾-in. gravel or stone. The concrete was vibrated with internal vibrators, using small vibrator heads where necessary. Accumulation of rubbish inside tank wall forms was overcome by leaving small openings at the bottom of the forms, and flushing the construction joint through pipes lowered inside the wall forms from the top. It was necessary to avoid wetting the wall forms to prevent washing off the retarding chemical coating. To provide good bond between old and fresh concrete, grout was first placed on top of the construction joint before placing the regular concrete.

There were many bronze fittings for each tank. Pockets were provided in the concrete walls for most of these which were later grouted in. A few of the fittings were placed directly in the forms before concreting.

REINFORCING

All tank walls and floors were doubly reinforced, both ways, both faces, with diagonal bars provided at all wall and floor intersections and at openings (Fig. 9 and 10). Steel was positioned to provide ¾-in. coverage. Tie wires

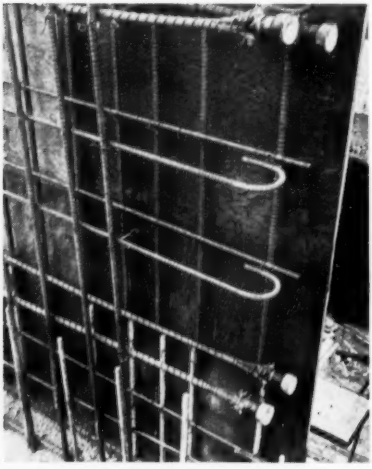


Fig. 9—Typical tank wall reinforcement with prestressing cables in place

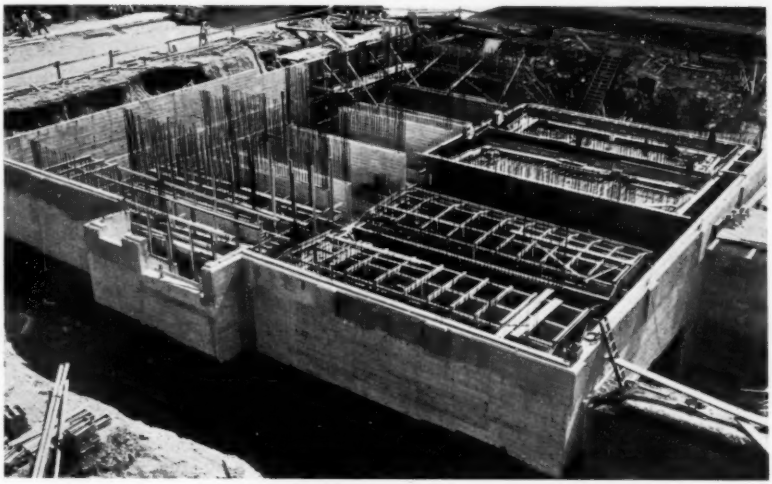


Fig. 10—Construction work for a six block tank building six stories high

or metal from slab spacers were not allowed to be exposed on the concrete surface. To avoid this and space the reinforcing correctly, small ¾-in. thick concrete blocks were precast with tie wires embedded in them and tied to the reinforcement, thus effectively preventing the steel from touching the formwork. Special metal spacers secured the steel correctly in the walls. For floor slabs, the small concrete blocks supported the bottom layer of steel, while for the top layer a continuous type of high chair resting on the bottom layer of floor steel was used. The top steel had not less than 1 in. below the finished floor. This required accurate inspection.

On the latest installation, an added feature was partial prestressing of longitudinal walls. These walls were partly prestressed to eliminate shrinkage cracks which usually develop during curing of the concrete and cooling of the building. Prestressing was accomplished by using prestressing cables of the correct length, replacing part of the standard reinforcement for shrinkage and temperature. The amount of prestressing was enough to permit a stress of approximately 100 psi. This installation is now under observation to determine the effectiveness of the partial prestressing.

GENERAL

As previously mentioned, a 10-ft wide operating aisle was provided between the tank blocks. The corridor slab was designed to rest on continuous concrete brackets placed monolithic with the tank walls. To avoid bond between the tank blocks and the corridor slabs, the tops of these brackets were finished smooth and painted with asphalt or other material, thus permitting independ-

ent movement of each block. Before applying the tank lining the concrete usually was heated with unit steam heaters to 90 F for a week or longer. This was done to eliminate all moisture in the tank walls and floor. Blowers provided thorough ventilation before and during the lining work. After completion of the lining work, all tanks were tested by filling them with water and calibrating them. The water test was made wherever possible by filling the entire block in checkerboard fashion to provide the most severe loading. White wall tile finished the exterior face of the tanks on the corridor side, providing a sanitary installation and a pleasing appearance.

Title No. 48-25

Conservation of Steel by Design*

By O. W. IRWIN†

SYNOPSIS

Conservation of steel is a live issue. This paper is limited to suggestions for saving steel in reinforced concrete design; it does not deal with the broader conservation possible through substituting reinforced concrete for structural steel.

SELECTING FLOOR SYSTEM

Some suggestions for saving reinforcing steel are obvious and need only be listed as *reminders*. Solid flat slabs for heavy live loads on relatively long spans when supported by a regular system of columns immediately come to mind. For light live loads on intermediate span lengths or for irregular column spacing one thinks of concrete joist construction. When you desire to save steel, beam and girder floor systems are avoided and maximum permissible depths of concrete are chosen.

Not only to conserve steel, but to obtain lowest first cost you design in tied rather than in spirally reinforced columns. It is an old adage that the cheapest reinforcement for columns is more cement and better graded aggregate if reduction in floor area is not too important a factor.

Considerable steel can be saved by careful attention to details. More bars are made full length than can be justified by efficient design. The 1951 ACI Code requires that only ¼ of the steel shall be full length in continuous construction and ⅓ in simple spans. Of course more field inspection will be required to assure accurate placing of ¾-length bars, but the ultimate cost would be decreased.

ELIMINATING HOOKS

Too many structural engineers still call for hooks. Rarely indeed is ample anchorage not provided by the bond strength of the new A305 bars. Richart's footing tests amply demonstrated the uselessness of hooks on the new bars. In pull-out tests a 20,000 psi stress in the bar is transferred to the concrete in less than 13 diameters of the bar, and a 30,000 psi stress in 15 or 16 diameters of bar. In flexure tests bond strengths have been recorded in the 1500 to 2000 psi range. With such strengths in bond, hooks are utterly useless.

*Presented at the ACI St. Louis Regional Meeting, St. Louis, Mo., Oct. 30, 1951. Title No. 48-25 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 23, No. 5, Jan. 1952, *Proceedings* V. 48. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than May 1, 1952. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, President, Rail Steel Bar Assn., Chicago, Ill.

It is estimated that hooks have been costing reinforced concrete construction between 20 and 25 million dollars per year when one adds to the initial cost of the hooks the increased cost of placing hooked bars plus the spoon-feeding of concrete through the many nests of curved bars. Engineers for U. S. Steel Corp. estimate a saving of over $100,000 in the foundations for the new Fairless Works. As reported in the Sept. 6, 1951, *Engineering News-Record*,

> Engineers took advantage of the American Concrete Institute's new building code provision allowing increased bond for deformed bars (*ENR*, March 1, p. 23) and consequent omission of many hooks. They used a high proportion of straight bars on the job. This cut down on bending costs as well as on the amount of steel required for the job's half-million yards of concrete.

Let's avoid this financial and steel wastage by relying on the new high, closely spaced deformations.

HIGH STRENGTH STEEL

Enough of the obvious, generally accepted methods of saving steel. How about the real economies possible through working the steel more efficiently, through using higher strength steel. That's where the emphasis is being placed. Witness the recent upsurge of interest in prestressed concrete and in applications of various forms of cold-worked steel.

A step in the direction of working steel more efficiently in tension was taken by the emergency specifications of the second World War. Liberalizing working stresses was the number one topic in the February, April and November 1942 ACI JOURNALS. R. L. Bertin sounded the keynote when he wrote,

> It is highly desirable that the production of high yield point bars be encouraged and that the use of correspondingly higher unit stresses be sanctioned by authoritative organizations.

But the designing fraternity was not quite ready for that step and insufficient conservation resulted because many designers artificially boosted their live loads. The emergency specifications were also deficient, in that they gave no recognition to the really higher yield point steels. Everything above 40,000 psi yield point was lumped into one general class with a 24,000 psi working stress.

In February 1951 *Engineering News-Record* sensed the increased interest in higher tensile stresses by an editorial reading in part,

> The old arguments concerning excessive cracking of concrete with higher steel stresses apparently do not apply to the A305 bars. Therefore, consideration should be given to permitting higher allowable tensile stresses for A305 reinforcing bars. In view of current shortages of steel, such a step is particularly desirable right now.
>
> In fact, it's quite possible that before long government agencies concerned with conserving steel may seek to establish higher working stresses for reinforcing bars. These agencies would most certainly be helped by any prior accomplishments in this direction by the ACI and other technical organizations. The ACI can aid the defense effort by approving the code revision permitting higher bond stresses and by starting work at this annual meeting on a code revision increasing tensile stresses for A305 bars.

HISTORICAL INCREASES OF WORKING STRESS

The evolution of working stresses over the last half century is interesting. In the first decade, ordinary flexural stresses were 12,000 or 14,000 psi in the steel and 450 psi in the concrete. About 1910 these were advanced to 16,000 in the steel and an average of about 750 in the concrete. Some fifteen years later there was a great deal of discussion, with considerable resistance from the structural steel interests, before 20,000 psi on intermediate grade and 18,000 on the structural grade of reinforcing bars began to be used. Our ACI Code has so stressed steel for about 23 years.

In the meantime, the mixing and placing of concrete has become a science, and concrete working stresses in compression have appropriately increased. In contrast with the 750 psi of 1910, 2250 psi today is not unusual, and 1500 to 1700 psi are daily occurrences. Why is it that the steel stress has remained stationary for twenty-odd years, whereas concrete compression has steadily increased with the increase in predicted strength of the final cured concrete?

"Predicted strength" is used advisedly. The actual strength of a batch of concrete is not known until it has been in place for 28 days. In contrast, through mill test reports, the strength of the steel is known before it is fabricated and shipped to the job. Is it consistent to work the concrete to 45 percent of its predicted ultimate crushing strength while limiting known high-strength steel to less than 20 percent of its ultimate tensile strength or to 40 percent of a guaranteed minimum yield point?

INCONSISTENCIES IN STEEL STRESSES

There is a further inconsistency in present working tensile stresses. The allowable stress in structural grade reinforcing bars is 55 percent of its minimum yield point. The intermediate grade stress is 50 percent of its minimum yield point, and the hard grade and rail steel grade are only worked to 40 percent of their guaranteed minimum yield point, or about 30 percent of the average yield point.

To repeat for emphasis, working stresses in concrete are functions of its ultimate crushing strength, whereas the yield point of the steel is treated as its ultimate. Between the yield point of the steel and its ultimate tensile failure, there is a wide margin of strength available to take care of unpredictable overloads, with a plastic yielding which warns of any impending failure, rather than the sudden crushing which characterizes failure in concrete.

Real conservation of steel in reinforced concrete design can be effected by making the working tensile stress a function of the yield point of the steel. A 60,000 psi yield point steel costs no more to manufacture than a 33,000 psi yield point steel. As a rule, mill test reports show yield points considerably beyond the specified minimum. It is, therefore, urged that tensile unit stress in longitudinal reinforcement be 60 percent of the guaranteed minimum yield point—20,000 psi in structural grade, 24,000 in intermediate grade, and 30,000 in hard grade billet or in rail steel under present ASTM specifications.

Two objections to this proposal have been voiced. One, so-called "increased deflection" and the other the possibility of "tension cracks."

Deflection

The deflection of a structural member is inversely proportional to the moment of inertia of the section. The effect on the "transformed moment of inertia" by reducing the steel area by ⅓ is insignificant compared to the proportion of the total moment of inertia provided by the concrete. There seems to be no particular worry about deflection when the compressive stress in the concrete is doubled as the crushing strength of the concrete is doubled. Does one have excessive deflection by stressing 4000 lb concrete to 1800 psi as against 900 psi on 2000 lb concrete? Of course the deflection should be computed; or "estimated," on critical members, and these higher stresses may make advisable a slight change in the rule of thumb methods of determining minimum depths of flexural members. That would be in line with the reminder earlier, that steel can be saved by increasing the depth of the members.

Tension cracks

With plain bars, or with poorly deformed bars, the crack fear is valid, but the new, high, closely spaced deformations prevent movement between concrete and steel to give enormously high bond strengths when the longitudinal variation of tension in steel is measured with strain gages. Concrete adjacent to steel begins to crack under the elongation which exists when the steel is stressed at about 6000 psi. The cracking is microscopic, the new deformations prevent the movement which would permit these microscopic cracks to accumulate into a visible crack. The maximum possible width of crack is limited to the elongation of the steel between adjacent deformations.

At a 30,000 psi stress in the steel, the elongation is 0.001 in. per in. Under the requirements of ASTM A305, the new high deformations must be staggered on opposite sides of the bar making a space of less than ⅓ in. within which this elongation might provide a crack in the concrete. A crack of ⅓ of 0.001 in. is a far cry from the width of 0.01 in. advocated by Amirikian in 1950 as a criterion beyond which the cracks should be considered deleterious.

In the July 1936 *Structural Engineer* it is recorded that the Washington State Highway Dept. exposed a series of cracked reinforced concrete blocks for ten years. Under the weather conditions of that region, it was found that crack widths as high as 0.02 in. did not induce dangerous corrosion in the steel. Thus author F. Thomas would double Amirikian's limit on "nondangerous" crack widths. Both of these "safe" limits are many times the anticipated ⅓ of 0.001 in.

Some research professors and engineers have even asked why make the ceiling 30,000 psi. It is their testimony that with the new A305 deformations, it is necessary to *pass* the elastic limit in the steel before obtaining noticeable cracks.

HIGH TENSILE STRESS PRECEDENTS

We in the United States have now reached the necessity for conservation which for years has characterized the economy of other countries. These recommended changes in tensile working stress are not steps in the dark. European engineers have led the way for years.

Abroad

The British Standard Code of Practice, CP114, adopted in 1948, in stating the "permissible stresses in reinforcement," provides that the "tension other than in helical reinforcement in a column or in shear reinforcements," may be "half guaranteed yield point but not more than 27,000 except in the case of prestressed concrete." That Code is more conservative on "tension in shear members in beams," in that the provision for permissible tensile stress for such shear members reads "half guaranteed yield point, but not more than 20,000." The foregoing quotations are from the British Standards Institute publication *The Structural Use of Normal Reinforced Concrete in Buildings*, and are applicable to *plain bars* with no deformations to control cracking.

In a personal letter last year, Professor Eivind Hognestad, University of Illinois, wrote the author:

> I have just been catching up on my reviews for the ACI. A major job has been to study brand new reinforced concrete specifications from Sweden and Denmark. I also came across an article by Lage Themner in the Swedish trade journal *Cement och Betong*, Nov. 1, 1950, entitled "Concrete Specifications in Some Countries." Mr. Themner compares these specifications: Denmark—1949, Finland—1946, Norway—1939, Sweden —1949, Germany—1943, and ACI—1947. He finds that the general trend of all are very similar; but with some surprise it is found that the "steel-country" U. S. A. is the *only* country that does not use *hard* grade steel economically. For deformed bars of hard grade, yield point over 50,000 psi, the Swedes and Danes use allowable stresses *over* 30,000 psi.

Reference to the original article discloses that the contrast between German and United States practices is emphasized in a graphic presentation of the working tensile stresses of the two countries. Germany with a minimum yield of 50,000 psi works the steel to 28,400 psi; the United States with a minimum yield of 50,000 works the steel only to 20,000 psi.

In the December 6, 1946 issue of *The Engineer*, Erling Reinius summarized the deformed bar situation in Sweden. According to this article the Civil Service Dept. of Sweden approved the new deformed bar in 1942 and established the regulations for its use. Incidentally, the minimum requirements on deformations were practically the same as contained in ASTM A305-50. The Swedes beat us by 8 years.

On these new deformed bars, at the very beginning of their use, nine years ago, a tensile stress of 28,500 psi was authorized, a bond stress of 300 psi was permitted, and the specification clearly stated "end hooks are not necessary." Under "Economic Results" Mr. Reinius states,

> At an inspection of several factory buildings, where the floors were loaded with heavy vibrating machines, practically no cracks in the concrete could be found. In most of these buildings the permitted steel stress was 30,000 psi, and end hooks were not used in any of them.

Mr. Reinius is an enthusiast for the new deformed bars and mentions particularly a dam and a water power station built with 550 tons of the new deformed bars, whereas the old type of bars under the old specifications would have required about 800 tons—conservation to the tune of 31 percent.

At home

We are not without sizeable structures designed to 30,000 psi in the United States. For over four years the Civil Works Division of the U. S. Army Engineers has been definitely committed to an increase in working stress of 50 percent whenever stretched and twisted bars, which have a minimum yield point in excess of 60,000 psi, are used.

Our own ACI Code allows 30,000 psi on welded steel wire fabric on which the bond stress permitted is the same as on deformed bars when the cross wires are not over 6 in. apart, a rather wide spacing. The ASTM minimum yield point for the wire in such fabric is 56,000 psi.

Several very large housing jobs bear adequate testimony to the crack control of the new deformed bars where the designers have been bold enough to use 30,000 psi on steel having a guaranteed minimum yield point of 50,000 and an actual yield point of around 65,000. Unfortunately too many of these who conserve steel by working it as efficiently as they work the concrete, whenever not prevented by municipal codes, do not care to have their names mentioned. As one remarked to the author when asking for permission to quote him, "Why should I reveal my trade secrets to those who do not understand reinforced concrete as well as I do."

To quote another friend, sometimes bold but always conservative,

> One outgrowth from the improved bar will be better structures through the closer spacing of much narrower cracks in tension concrete. This is of advantage not only in concrete barges, tanks and similar vessels but also in rendering much less unsightly the conditions of highly stressed negative moment zones around supports.
>
> Beyond this, still further progress is envisioned, possibly coming more slowly; but with improved bars affording better resistance to diagonal tension, some change is likely in that field; and with bond and diagonal tension well taken care of, it may be possible to raise working stresses in tension and also to use ultimate design methods.

Or a research professor:

> It appears to me that your bar is an answer to a reinforced concrete engineer's dream, in that it seems to prevent *any* slippage due to high bond stresses. In previous tests we had used the old type bars and tension cracks developed, spaced 5 or 6 in. apart. The new bars prevent this cracking within the elastic limit of the steel.

CONCLUSION

The present tension limits were most wise with the old deformations. End hooks did not control the cracking of the concrete at points of maximum tension. With the new A305 deformations, further economy and real conservation of steel can be effected by working the steel in proportion to its tensile strength, just as engineers have for years established compressive stresses in concrete. And the high strength steel will not cost a penny more in dollars per ton.

Discussion of a paper by O. W. Irwin:

Conservation of Steel by Design*

By K. HAJNAL-KONYI, EIVIND HOGNESTAD and AUTHOR

By K. HAJNAL-KONYI†

Mr. Irwin has pointed out the inconsistency of present working tensile stresses, whereby the allowable stress in structural grade reinforcing bars is 55 percent of its minimum yield point, in intermediate grade 50 percent and in hard grade 40 percent. He recommends "that the tensile unit stress in longitudinal reinforcement be 60 percent of the guaranteed minimum yield point—20,000 psi in structural grade, 24,000 psi in intermediate grade, and 30,000 psi in hard grade billet or rail steel under present ASTM specifications."

The writer should like to go one step further. In assessing working stresses, one should consider not only deflection and cracking but also the warning obtainable before failure. This principle is acknowledged in the ACI design provisions for columns‡ which call for a factor of safety 25 percent greater for tied columns than that for spirally reinforced columns, in view of "the well-known sudden and violent type of failure observed in the tied columns." "Sudden and violent" failures do not occur in well-designed reinforced concrete beams (*i.e.*, when shear failure is excluded) but the question of warning should nevertheless not be overlooked.

In the case of deformed bars complying with A 305-50T the crack control is so good that with 1 in. diameter bars of a yield point of 40,000 psi the yield point can be reached with cracks not exceeding a maximum width of 0.006 in. With 1 in. diameter bars having a yield point of 50,000 psi, the maximum crack width may not exceed 0.01 in. immediately before yielding begins. The great advantage of using large size bars of a minimum yield point of 60,000 psi is that better warning of the approaching danger is given since the cracks are much wider before the steel can yield. In the case of small size bars ($\frac{1}{2}$ in. diameter and less), even with a yield point of 60,000 psi, yielding may occur when the cracks appear to be quite harmless. For bars of such small size a minimum yield point of 70,000 psi would be desirable.

It may appear paradoxical but one cannot escape the conclusion that *the greatly improved crack control of deformed bars calls for a reduction rather than an increase of working stress-yield point ratio in the case of structural grade*

*ACI JOURNAL, Jan. 1952, *Proc.* V. 48, p. 373. Disc. 48-25 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 24, No. 4, Dec. 1952, Part 2, *Proceedings* V. 48.

†Consulting Engineer, London, England.

‡Richart, F. E., "The Structural Effectiveness of Protective Shells on Reinforced Concrete Columns," ACI JOURNAL, Dec. 1946, *Proc.* V. 43, p. 353.

steel. It would be consistent with the concept of safety *to increase this ratio with increasing guaranteed minimum yield point, i.e.,* to reverse the present ACI Code. This would apply up to a stress where the maximum crack width at working loads becomes again the limiting factor, *i.e.,* well beyond 30,000 psi, even with large size deformed bars.

A structure designed for 30,000 psi in which hard grade steel of a yield point of 50,000 psi is used is "safer" than a structure designed for 18,000 psi with structural grade steel of a yield point of 33,000 psi in which the steel may begin to yield at a stage when no visible cracks occur. However, as stated above, only a yield point of 60,000 psi in the case of large size bars and a yield point of 70,000 psi in the case of small size bars is a safeguard for adequate warning before failure, irrespective of the permissible stress adopted.

By EIVIND HOGNESTAD*

The writer agrees that increased working stresses for high strength reinforcement should be seriously considered at the present time. In expressing this point of view, however, he would like to make it clear that such an increase cannot safely, in his opinion, be carried out simply by substituting a higher stress for the allowable value of 20,000 psi given in ACI 318-51, 306(*a*).

A building code such as ACI 318-51 is a complex document, which has gradually been developed through half a century of research and practical experience; it has been influenced also by the judgment of several generations of concrete engineers. Any proposed change of this code must therefore be considered in terms of its possible effects on the code as a whole. To change certain details without duly considering the effects such changes may have on other parts of the code may lead to unfortunate results. Similarly, if two different codes are compared, for instance with respect to allowable stresses in the tension reinforcement, other differences in the two codes must also be considered.

As pointed out by the author, many European codes allow considerably higher stresses for high strength tension reinforcement than the 20,000 psi permitted by ACI 318-51. It is believed to be important, however, that other differences than reinforcement stresses exist between European and American practice. Considering for instance the Swedish Code S.B.-1949, at least the following additional differences appear.

1. The Swedish code differentiates between structures essentially subject to static loads such as common buildings and hydraulic structures, and structures subject to dynamic effects, such as bridges and most industrial buildings. Allowable stresses of about 30,000 psi are permitted for deformed reinforcement with a minimum yield point of about 55,000 psi only for structures essentially subject to static loads. No such differentiation is made in ACI 318-51 with respect to the type of loads.

2. Allowable stresses in the Swedish code further depend considerably on the quality of the construction work, three classes being considered. The

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high steel stresses are permitted only if construction is carried out under strict control, with well experienced supervisors, and employing only skilled concrete workers. For the type of control given as a minimum by ACI 318-51, the Swedish code permits only about 16,000 psi for tension reinforcement.

3. High allowable steel stresses in the Swedish code are permitted only for concrete strengths over 3500 psi. No such minimum is given by ACI 318-51.

4. The Swedish code calls for web reinforcement designed for the total shear, no shear being considered carried by the concrete, wherever web reinforcement is at all necessary. For T-beams a minimum amount of stirrup reinforcement is specified throughout the length of the beams regardless of the magnitude of the shearing stresses. It is apparent that this Swedish design of web reinforcement is much stricter than American practice. In recent exploratory tests the writer has found that beams designed in accord with ACI 318-51, that is $f_s = 20{,}000$ psi, $f_c = 0.45\ f_c'$ and $v = 0.03\ f_c'$, quite commonly fail in diagonal tension at nominal shearing stresses v below $0.05\ f_c'$ without developing the yield point of the tension reinforcement. As the shearing strength of beams is known to decrease with decreasing percentage of longitudinal reinforcement, an increase of the allowable stress for high strength reinforcement would render this situation even worse. The high strength of the reinforcement will in this case provide no "margin of strength" whatsoever.

5. High allowable steel stresses in the Swedish code are permitted only for bars with a diameter less than $\frac{5}{8}$ in., and the allowable bond stresses are of the order 0.05 to $0.07\ f'_c$ for deformed bars as effective as the American ones. These low bond stresses as compared to the value $0.10\ f'_c$ used in ACI 318-51 are particularly important to control the increased cracking tendency resulting from the use of high allowable tensile steel stresses. The author asserts that elongation of the reinforcing bars at a flexural crack must take place within the $\frac{1}{3}$-in. spacing of the lugs. If the bond strength really was high enough to permit such a transfer of stress, flexural cracks in the concrete should form at a spacing of about $\frac{2}{3}$ in. while tests with bars well passing ASTM A 305-50T show a spacing of at least 6 to 8 in.

The writer feels consideration of increased working stresses for high strength reinforcement is timely and needed. He recommends, however, that such a change be considered with great caution, particularly with respect to the possible effects on other than flexural modes of failure such as diagonal tension failures.

AUTHOR'S CLOSURE

The discussions of Messrs. Hognestad and Hajnal-Konyi are appreciated by the author; their comments are timely and pertinent.

The report of the Building Research Advisory Board on Conservation in Building Construction dated June 30, 1952, recommends that ACI Committee 318 "be encouraged to expedite completion of its current study and recommendations in regard to the increase in the allowable tensile stresses of steel

reinforcement bars" (paragraph 3.3—p. 77). The points raised by Messrs. Hajnal-Konyi and Hognestad merit consideration by that committee..

For over ten years, forward looking engineers have been using higher strength steels more efficiently, thus encouraging its production. Mr. Hajnal-Konyi adds a new thought when he emphasizes the logic of using a higher percent of the yield point as a working stress on large bars, or the advisability of requiring a higher yield point on small bars.

The author also agrees with Professor Hognestad that the problem is more complicated than the mere substitution of a 60 percent formula in Section 306(*a*) of ACI 318-51. There should doubtless be a minimum concrete strength when the steel is stressed to 60 percent of its minimum yield point; perhaps 3500 psi is a good figure. For lower concrete strengths a lower percentage might be adopted, but there is no valid reason why a 65,000-lb yield point steel should be limited to the *same* tensile stress as a 40,000-lb steel even in lower strength concrete.

In the author's opinion, provision for dynamic loads should be covered by a *reasonable* impact percentage addition to the static load, equally applicable to tension, to compression and to shear. Please note the "reasonable," as many recent tests on railroads structures indicate their designers went "hog-wild on impact."

Perhaps a small part of the tensile steel saved by the author's proposal should go back into the design as web (shear) reinforcement. Let's put the steel where it's needed. Some highly efficient recent designs have nearly doubled the ordinary stirrup steel and greatly decreased the over-all cost by using straight bars only to provide for flexural tension.

AREA has published the findings of extended strain gage measurements on reinforced concrete structures under modern traffic on three midwestern railroad systems. Under both diesel and steam locomotives at speeds of 5 to 80 mph there was not recorded a single tensile stress in a reinforcing bar as high as 3000 psi from live load plus impact. Many a designer will be surprised at the low impact addition from 80 mph speeds, and the even lower longitudinal forces when applying emergency brakes.

High strength steel, pound for pound, costs no more than low strength structural grade. With the new crack-controlling deformations, it should be made "legal" to use high strength steel more efficiently.

Title No. 48-26

Automatic Jacks Speed Sliding-Form Construction*

By DAVID F. STOUT† and ROBERT E. WILDE‡

SYNOPSIS

Electric jacks which leveled automatically, and threaded steel plugs for joining jack rods, offered advantages in sliding-form construction of a grain elevator. Notable were speedier construction, simpler erection procedure and minimum labor crews.

Essential factors in sliding-form construction are reviewed and construction procedures described.

INTRODUCTION

Sliding forms for construction of tall grain elevators are not new, but they are spectacular in the speed with which construction moves. New techniques, first used by Jack Construction Co. in 1949 and used in the construction of a 22-tank grain elevator south of Ottawa Lake, Mich., offer greater speed and simpler construction with sliding forms than previous methods. Notable innovations were the use of automatically leveled electric jacks, and a method of joining jack rods which made possible continuous operation of the jacks and simpler erection procedures.

The elevator described is to be built in two parts; the first part consists of 14 tanks and a workhouse; the second, eight additional tanks separated from the larger structure (Fig. 1). The 21 ft 2 in. diameter tanks with 7 in. walls are 117 ft high with a headhouse 73 ft above the roof slab. Using a minimum labor force, the sliding-form construction of the major portion of the project used 3000 cu yd of concrete and was topped-off in 206 hours.

GENERAL CONSIDERATIONS

The use of continuously moving forms is an economical method of building a tall reinforced concrete structure which has more or less the same shape throughout its height. Notable advantages are speed of construction and a smooth surface finish since there are no horizontal joints. Sliding forms allow construction to proceed about four times as fast as with fixed forms. The continuity of concreting operations, centralization of mixing plant and materials, and easy access to all work from the deck or platform tend to produce greater working efficiency than with usual monolithic construction.

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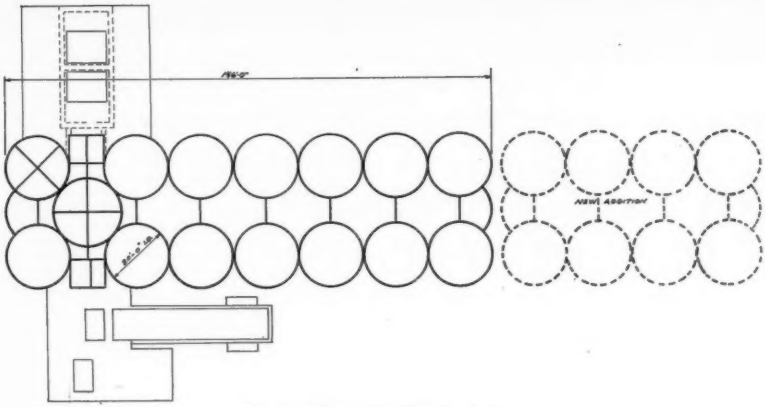


Fig. 1—Plan view of grain elevator

If full advantage is to be taken of sliding-form construction, the structure should be at least 50 ft high. The higher the structure, the more economical are sliding forms.

Essential factors

Essential factors in sliding form construction are:

(1) All parts of the forms must move upward at the same time and there must be no dragging of one section. Forms must be made accurately and jacking operations supervised closely, or, as in this case, fully automatic.

(2) Lateral supports must be provided for the forms.

(3) Walls should not be less than 6 in. thick because the depth of plastic concrete in the forms at any time is limited. If the weight of the concrete is insufficient, the concrete may be lifted by friction as the forms move upward, leaving a "fall-out" (hole) in the wall.

(4) Best results are obtained if sliding-form construction is continuous. Stopping and restarting results in forms bonding to the hardened concrete, which often lifts the top layer of concrete when the form is broken loose.

(5) More than any other method of construction, sliding forms depend for success on a uniform and consistent supply of materials from mixing plant to working deck. It is important that there is sufficient storage space for all materials on the job. If possible, there should be stand-by equipment available for mixing, hoisting, etc.

(6) Reinforcement should be as simple as possible and with a minimum of tying which would retard the work.

(7) Extraneous features and fittings should be reduced to a minimum. The faces of the walls should be plain throughout the height of the structure, although changes in thickness can be allowed for by setbacks or tapering.

All the basic principles of concrete construction by other methods apply equally well, with added emphasis on job planning and scheduling for continuous operation.

CONSTRUCTION PROCEDURE

Construction operations usually follow this pattern:

(1) Foundations built while forms are being made, (2) hoisting equipment, plant and materials placed, (3) sliding forms, reinforcement and jack rods placed for initial pour, (4) continuous sliding-form construction, (5) topping-off, roof slab concreted and forms removed, (6) headhouse, interior beams, slabs, extra features added and (7) grain processing equipment installed.

The foundation for the Ottawa Lake elevator was placed well in advance of sliding-form operations. The pits were excavated to within 18 in. of bedrock, the muddy sand consolidated, and the foundation slab and pit walls placed. Specifications called for 2500-lb concrete, although 28-day tests showed some of the concrete testing as high as 5100 psi.

The hoist tower was erected to its full height before wall construction started and was guyed securely. As concreting progressed and the guys fouled the sliding forms, the tower was attached to the tank walls and the guys moved up. The mixer discharged directly into the hoist skip, which in turn discharged into a hopper on the working deck; concrete was placed by buggies.

Concrete

The coarse aggregate, sand and cement were stockpiled in a semicircle about the batching plant, which fed directly to the 16-cu ft mixer.

Specifications called for 3000-lb concrete in the tank walls. A 3-bag mix was used with the following proportions: No. 4 stone, well graded, 970 lb; sand, 800 lb; cement, 282 lb. About 1 oz of Darex per bag of cement was added at the mixer to give about 5 percent entrained air in the mix.

Weather and climate determine to a great extent the type of cement that will serve best. Originally, new cement that is still moderately warm gives fast enough set for sliding-form concreting. In winter, high-early strength cement has been used to give quick set; in tropical climates it is often necessary to use slow-setting cement.

Reinforcement

The 1-in. jack rods were the only vertical reinforcement in interior walls of the tanks. For outside walls, vertical reinforcement included, in addition to the jack rods, ½-in. round bars at 2 ft centers; horizontal reinforcement consisted of ⅝-in. round bars at 12 in. centers. For the latter, many designers prefer to use ½-in. bars at 8 in. centers. There were no hooks on the circular reinforcing bars; hooked bars were placed only in slabs and beams. At corners and where tanks joined, small sections of bars were bent to follow the wall outline. Bars were old-type deformed, which are now classed as "plain bars" to differentiate them from the new-type deformed bars meeting ASTM Specification A305.

The steel workers threaded the circular horizontal reinforcing into the wall as sliding-form operations progressed. Usually the concrete supported

the bars at the correct position without tying. Tying was kept to a minimum so that it would not delay concreting. Occasionally it was necessary to tie a circular bar to a jack rod or vertical reinforcement if the bar was sprung. Otherwise the bar might lodge against the form and damage the wall as the form moved upward, possibly causing the forms to stick.

Formwork

Careful workmanship is necessary on formwork for sliding-form construction. The staves used on the 4 ft high circular forms were dipped in paraffin oil prior to assembly. The outside wall forms were 4 ft 6 in. high, the extra 6 in. being used as a splashboard to protect cement finishers on the scaffolding below.

The wales were lighter than many used on previous sliding-form jobs; cross bracing between the walls also was reduced to a minimum, which reduced the over-all weight of the platform.

The 4 in. wide and ¾ in. thick vertical staves were straight-grained and free from knots; they were nailed to upper and lower circular wales or chords. The composite top and bottom wales were 2 x 6 in. planks with segmental 1 x 6 in. boards, cut to follow the circular outline, fastened on the top and bottom of the planks with joints staggered (Fig. 2).

Diagonal bracing between top and bottom wales was not considered necessary except on each side of a yoke. This was sufficient to keep the forms from sagging.

To assist the sliding action, the forms had a slight draft. For outside walls, the outside form was plumb but the inside form had a 3/16 in. batter in 4 ft. There was a 3/16 in. batter on both forms for the inside walls. Thus in a 4 ft high form, the concrete got its shape about 21 in. up the form. The bottom part of the form was a little wider than the specified 7 in. wall thickness, the top a little smaller. By the time the wider part of the form passed over the concrete, the concrete had begun to set and retained the 7 in. thickness.

To prevent the forms from losing their circular shape, internal forms were braced by two timber trusses across the tank attached to the inside wales

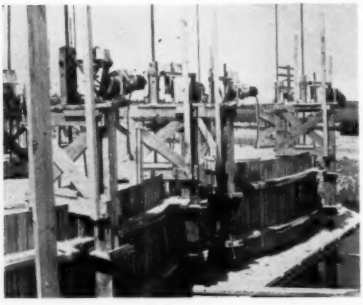


Fig. 2—Platform weight was reduced by using lighter wales than usual and reducing cross bracing between wales. Wales were comprised of planks and boards cut to fit the circular forms; joints were staggered. Yoke legs extended down along forms to aid alignment and to keep yoke vertical. Lift rods on each side of the jack rod were attached to the top transom of the yoke and to both top and bottom wales

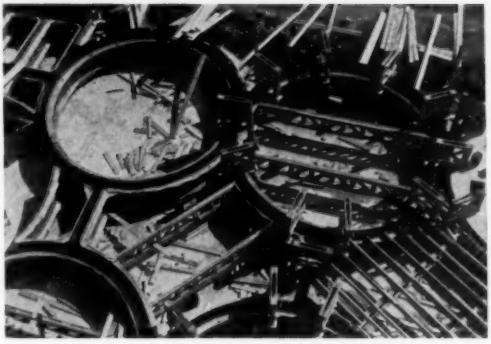


Fig. 3—Circular forms were braced with interior trusses. Note the yokes already in position and attached to the wales

(Fig. 3). The tanks were then decked with 1 in. boards nailed to 2 x 6 in. joists. The ends of the joists rested on the top circular wales and the two bracing trusses. This timber decking formed the working platform and later the bottom form for the roof slab.

Since the forms swell once wet concrete is placed in them, the staves were kept between 1/16 and 1/8 in. apart.. A simple check on this spacing was to slide an 8-penny nail between each stave.

Jacking

The jacks (Fig. 4a and 4b) used on the Ottawa Lake elevator were of the newest type, electrically operated and automatically leveled. Approximately 180 jacks, each capable of lifting 40 tons and spaced about 7 ft apart, were used. Before lifting began, the jacks were leveled by a Dumpy level;

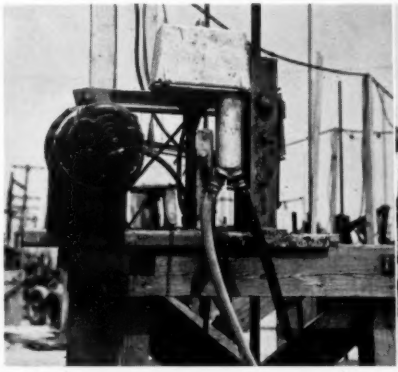


Fig. 4a—A 1/12 hp electric motor (left) operated the jack. The "brain box" (center) containing float valve and mercury switch was connected to the master tank by rubber hose for leveling control

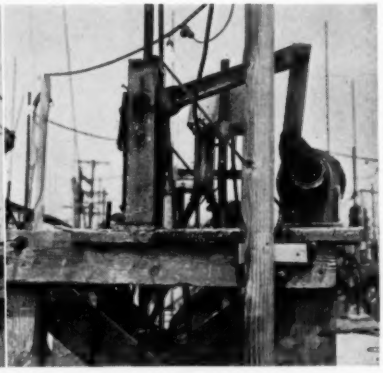


Fig. 4b—The electric motor operated the lever arm and jaws as the jack climbed up the jack rod. Note lift rods on either side of jack rod

from then on the leveling was automatic. Jacking operations were handled by one master mechanic and an oiler; whereas screw jacks would have required larger operating crews to give the jack about ¼ turn every minute at a signal from the foreman. Also, with screw jacks, it would have been necessary to de-clutch the grip on the jack rod, raise the jack and take a new bite on the jack rod every time a jack reached its limit of travel.

Optimum speed of lift was determined by weather conditions, efficiency of plant and workmen, special forming and other day-to-day variables. Maximum lift was 10½ in. per hour in increments of about 2 in. every 12 min. Forms moved too fast will not give the concrete a chance to set and it will slump out the bottom of the forms. Forms moved too slow cause the concrete to adhere to the side of the forms and to be dragged up with them, making holes in the wall and displacing reinforcement.

Each jack was fixed securely to a yoke attached to the forms and platform; the jack clutch or jaw gripped the jack rod embedded in the concrete. Operation of the jack raised the yoke and everything attached to it which resulted in sliding the forms past the concrete already placed.

The yokes consisted of two vertical legs connected by transoms. An aluminum yoke is in the planning stage which would reduce the weight of the moving platform considerably. Yokes kept the forms correctly spaced for wall thickness and through their lift rods transferred the weight of the forms and platform to the jack rods (Fig. 2). The legs of the yoke kept the forms aligned.

Leveling

The principle that water in an open system seeks its own level was used to control levels of the jacks during operation. Float valve cylinders in the "brain boxes" at each jack were connected in series by rubber hose to a master tank (Fig. 5).

When the form was to be raised, the master tank was lifted between 1 and 2 in. After waiting about 5 minutes for the water in the float cylinder to become level with that in the tank, the operator pushed the button starting the $_1{}^1{}_2$ hp electric motor which operated each jack. Upon being raised to a level where the float valve in the "brain box" rose to the top of the water, a mercury switch turned off the electric motor operating the jack. Each jack did not necessarily stop at the same time, especially if the jack was slipping, but continued until the platform and jack were level. Checking with a Dumpy level, or by complicated measurement was unnecessary; the operator knew that when the last jack shut off, the platform was level.

JACK RODS

Jack rods

Another labor- and time-saving feature of the project was the method of joining the 1-in., medium steel, hollow jack rods. The rods were 16 ft long, alternate rods being cut to 8 ft to start so that all rods would not have to be joined at the same time.

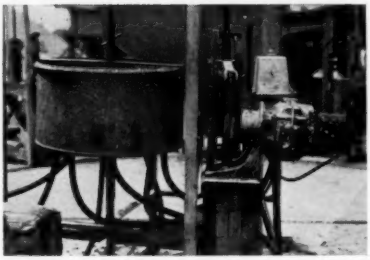


Fig. 5 (left)—Master water control tank for leveling operations was connected to the float valve cylinder at each jack



Fig. 6—Threaded steel plug used to join hollow jack rods

A threaded steel plug, 2½ in. long, was used to join the jack rods (Fig. 6). The plug was slipped into the bottom rod and given a few turns; the top rods being added the same way and turned until the joint was snug. The plug gives greater rigidity than past methods and offers the added advantage that jacking can be continuous past the connection. Formerly, a pipe sleeve about 6 in. long, dented in the middle to prevent it from sliding down the jack rod, was used. Often, when using screw type jacks, the jacks had to be de-clutched and lifted over the sleeve, a time-consuming operation.

The jaws of the jack bit into the jack rods to such an extent that small deformations were left, slightly improving bond between the rod and concrete.

Wall construction

With the plant in readiness for continuous operation and forms built, the reinforcement for the first 4 to 5 ft of wall was fixed and the forms erected for concreting. After the circular forms for walls were set up and leveled, the yokes were positioned and attached to the wales (Fig. 7); the jack rods were fitted in the yokes, jacks attached, and the deck boarded over.

Before any concrete was placed, a thorough check was made to see that the top of the forms was level to ensure vertical walls.

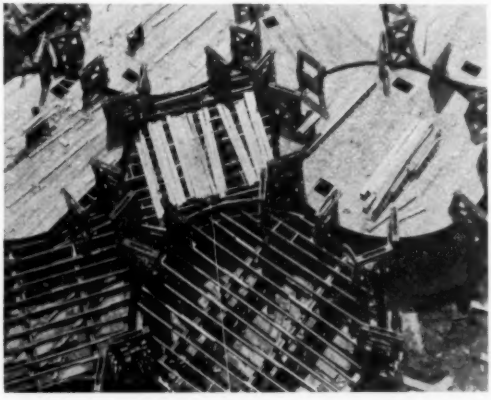


Fig. 7—Yokes were positioned about 7 ft apart around the tank circumference, closer at junctions of circular tanks and star-shaped interstices. They were attached to form wales, and the decks then boarded over

The spectacular part of the project, the wall construction, started at midnight, Monday, June 11. The full depth of forms was concreted all around the structure before the forms were gently eased upward. This first movement of the forms required caution since the forms must be started level and the jack rods receive their most severe strain at this time. The forms were moved upward in about ⅛-in. lifts. This rate increased as the job proceeded.

Since the first few feet go necessarily slow, by starting at midnight the forms were raised to the height where beam pockets and keyways were to be installed for bin bottoms by about 7 a.m. This meant that the boxes forming openings for the beams could be placed in daylight, just after a new shift came on the job.

Interior walls were started or stopped at any elevation, according to plan, while the forms moved upward. Stirrups were extended down from the walls to tie the slab, which was concreted later, into the walls (Fig. 8).

For setting beams, boxes were placed between and located in the forms at the correct position (Fig. 9); the forms moved past them smoothly. After the walls were concreted, the boxes were removed, and the beams and inside slabs formed and concreted (Fig. 10). It is possible in sliding-form construction to form these beams and slabs and concrete them as the job moves upward. However, fewer men are required when the walls are built first and

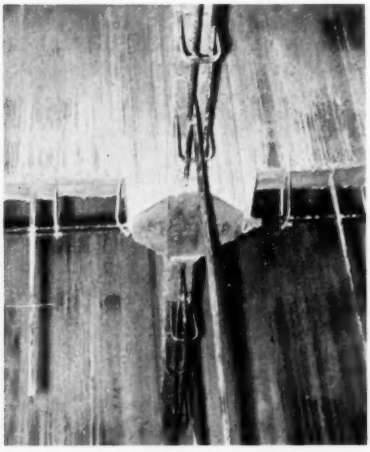


Fig. 8—Interior walls of workhouse were started about 15 ft up from bottom slab. Stirrups extended below the wall to tie in the future floor slab. Jack rods extended to the mattress slab during wall construction but were cut off when the slab was formed



Fig. 9—Box in form left opening for beam in bin bottoms which was concreted after the walls were completed



Fig. 10—Forming interior beam and slab. Note keyway left in wall during sliding-form construction



Fig. 11—(June 11) Rate of sliding forms averaged 8 in. per hour

small crews can put in interior features afterward. Steel window sash, door jams, etc., were set in similar fashion. The vertical reinforcing bars were cut to the exact elevation and the frames set in place on them so that the moving-forms would slide by.

Once past the first 10 ft (Fig. 11), the rate of rise increased to about 6 in. per hour at night and as the day warmed up it was possible to get about 10 in. per hour, or an average rate of 8 in. per hour.

On Wednesday, June 13, a steady rain slowed operations to 6 to 8 in. per hour during the heaviest downpours, but operations continued with little difficulty. During the rain the forms were jacked only about 1 in. at a time. By Thursday, June 14, the tanks were up 35 ft (Fig. 12) and the average rate of placement had risen to 10½ in. per hour. Operating three shifts, concreting was continuous through each night (Fig. 13).

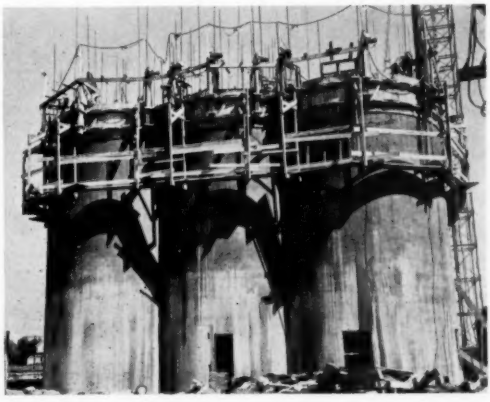


Fig. 12—(June 14) Rate of sliding forms had increased to 10½ in. per hour



Fig. 13—Night concreting was well illuminated and average rate of sliding forms compared favorably with daylight operations

By 7 a.m. Tuesday, June 19, the tanks had risen 100 ft (Fig. 14). The tanks were topped-off at 2:30 p.m., Wednesday, June 20—8 days and 14 hours to place 3000 cu yd of concrete in 15 117-ft tanks. In fact, the tanks would have been topped-off earlier if the mixer hadn't broken down which necessitated hauling ready-mixed concrete about 6 miles. (The ready-mix plant had been in stand-by readiness for such an emergency.) Ready-mix

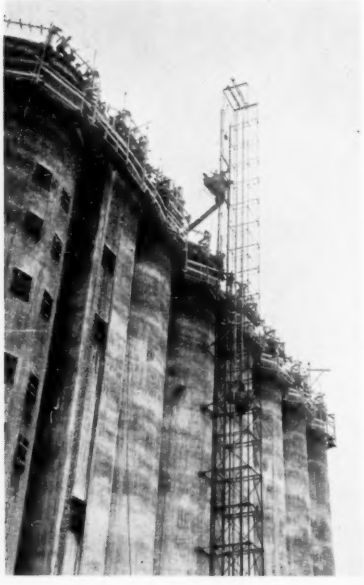


Fig. 14—(June 19) Over 100 ft high and almost to the top

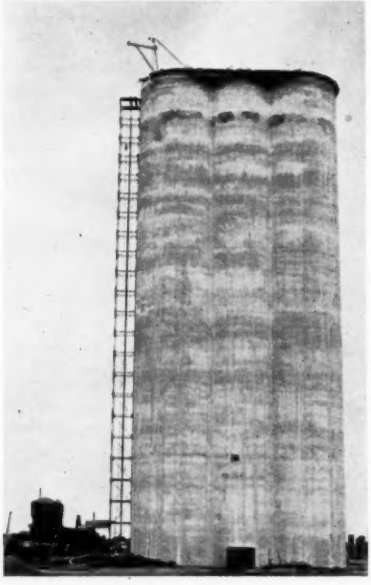


Fig. 15—(June 30) Walls have been "topped-off," exterior wall forms removed, and roof slab formed

concrete was difficult to handle since by the time it was unloaded and hoisted to the distributing hopper on the platform it had begun to set and get stiff.

During construction, the forms were checked periodically by walking around the scaffolding hung below the deck for the finishers. If the bottom of the form was so snug against the concrete that the finger tips would not go between the form and concrete, it was an indication that the concrete was too soft and exceeding the 7 in. wall thickness. This could be corrected by jacking at a slower rate and by checking the concrete to see that the specified slump was not exceeded.

Few problems arose, and those that did were solved by common sense and construction "know-how." Early in the project a spreader got caught in the wall between a jack rod and door jam under one of the yokes. As the form moved upward, it moved the concrete up with it, leaving a small hole. The hole was not noticeable until the form had risen past that spot. To repair, sheathing was put on both sides of the wall beneath the form and tied together; the spreader was pulled out and the concrete above was knocked down and more placed on top. Since the concrete was still green, it knitted with the rest of the wall and was smoothed off by the finishers.

Fortunately there were no "fall-outs," a soft spot in the concrete which slumps out after the form passes or a spot where the concrete sticks to the side of the forms and lifts with the form, leaving a hole.

During construction one of the forms began to sag below a dummy yoke which would have given a reverse batter to the wall forms, with disastrous results. (The dummy yoke is used as additional support to hold the forms at wall thickness but there is no jack attached to it.) A 4-ft channel iron was sunk in the wall below the yoke as a shoe to support a jack rod. A jack was attached to the yoke and rode almost 12 hours before it picked up the load, to ensure that the green concrete was sufficiently hard to support the jack rod and its load.

Finishing

For finishing, external scaffolding was suspended from the lower wales of the forms (Fig. 12), giving the finishers free access to the outside wall surface as the platform moved up. The concrete exposed under the forms was only 6 to 8 hours old, hard but green and ideal for finishing. The surface was finished with a rubber float and then a bristle brush.

By finishing the surface as the forms moved upward, it was not necessary to use a waterproof coating on the walls. The inside walls, which were not finished as the forms moved up, were finished where desired with a cement slurry. The walls were wetted and the slurry rubbed in with rubber floats.

No special precautions were taken for curing; however, for better insurance against crazing of wall and slab, it would have been advisable to run a stream of water over the slab and down the tank sides.

Topping-off

When the tanks reached a height of 117 ft, the outside circular wall forms

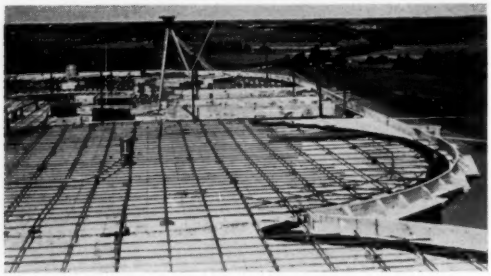


Fig. 16—Forming roof slab

were removed (Fig. 15) but the inside forms were left in place to support the platform that served as the bottom form for the roof slab. While the concrete at the top of the walls was still green, 22-in. long, sharp-pointed reinforcing bars were driven under the wales and through the concrete to support the form. The jacks, yokes and accessory equipment were removed and the platform patched to form the bottom of the roof slab (Fig. 16).

With the outside wall forms removed, a 4-in. side form was attached to the outside edge of the platform. Wood braces, attached to the slab form, rested against the tank walls to keep the form aligned with the walls. Where it was necessary to tie into the wall, the contractor used an explosive powered tool to drive the studs through the wood and into the concrete.

The roof slab is 6 in. thick at the headhouse and along the center of the structure, tapering to 4 in. at the outside of the tanks.

To remove the platform and forms inside the tank after concreting the slab, and for finishing interior walls, an interior scaffold was hung from the completed slab. Holes were drilled in the form and before concreting, bottles were placed with their necks extending through the holes. After the slab was set, the bottles were broken and cables slipped through the holes to hold interior scaffolding.

After the slab was finished, the headhouse, which rose another 73 ft above the roof slab, was formed and concreted. The remaining eight tanks will follow the same procedure and will be located 3 ft from the present tanks, a precautionary measure in case of unexpected settlement.

ACKNOWLEDGMENT

The structure is being built for Michigan Elevator Exchange, Lansing, Mich., by Jack Construction Co., Inc., Kansas City, Kans. Co-author Stout served as project superintendent; Tom Carpenter was assistant superintendent. A. J. Lindquist, Chicago, was consulting engineer.

Title No. 48-27

Correlation of Shrinkage and Curing in Concrete Masonry Units*

By HARRY W. EASTERLY, Jr.†

SYNOPSIS

Need for a specification defining maximum shrinkage in concrete masonry units has long been recognized. The present specification which limits the amount of moisture in the unit when it is laid is inadequate and cannot be enforced. This paper shows how use of the British specification which limits shrinkage rather than moisture content in the block was applied to block cured by high-pressure steam and by high-temperature steam. High-pressure steam cured blocks shrank only about half as much as those cured by high-temperature steam. The literature on high-pressure steam curing is briefly reviewed and the British standard method of measuring drying shrinkage is shown to be applicable to masonry units manufactured in this country.

INTRODUCTION

The continued cracking of concrete masonry walls is proof enough that the present 40 percent limit on moisture in blocks does not properly protect the purchaser. Since in most cases it is impractical to store blocks under cover immediately before laying, the chance of their absorbing more than 40 percent of total moisture content is great. A number of tests have proved that most of the shrinkage which takes place in a block between saturation and oven dry condition occurs *after* the block has been reduced to 40 percent moisture. It seems obvious, therefore, that the ideal specification to prevent shrinkage cracking is one which measures actual shrinkage in the masonry unit from its maximum to its minimum moisture condition. This is exactly what the British standard test (Appendix 1) does, using saturation as the maximum and 17 percent relative humidity at 122 ± 2 F as the minimum moisture condition. In practically no case would a block be subjected to a more severe drying condition than this. On the other hand, in tests in which blocks have been dried in an oven at 212 F and not subjected to controlled humidity conditions, there is evidence that erratic chemical changes take place which destroy the value of the tests. Furthermore, a long time is needed in bringing the blocks down to an "air dry" condition before they are put in an oven.

The inadequacy of present specifications is borne out by the statement of D. E. Parsons, a member of the committee which adopted the Federal Specifications for Hollow Concrete Masonry Units published in 1931:

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†Member American Concrete Institute, Executive Vice-President, Concrete Pipe & Products Co., Inc., Richmond, Va.

When the first specification was written, the Technical Committee was aware of the shrinkage problem, but it lacked the information needed as the basis for the preparation of an adequate requirement and suitable testing methods relating thereto. We were also conscious of the fact that manufacturers at that time had no facilities for drying the units. The limit on moisture content of 40 percent was considered severe because very few of the products then being delivered to the site of construction conformed to that limit. On the other hand, those who drafted the specifications were aware that the 40 percent limit might not prove to be an adequate protection to the purchaser. We had hoped that within a comparatively brief time of two or three years, the technical information needed for the formulation of satisfactory requirements would become available.

I think that the first ASTM Specification, which included the 40 percent limit, also was published in 1931 and that its authors shared the hopes and misgivings of those who prepared the Federal Specification.

The British government recognizes the importance of controlling volume change of masonry units by limiting actual shrinkage of the unit itself to 0.04–0.06 percent. They have controlled volume change by reduction in cement; the resulting lean mixes, while satisfying the desired shrinkage requirement, drastically reduce strength. For instance, for precast cinder partition blocks, the average compressive strength requirement is only 150 psi. The maximum requirement for any block, load bearing or non-load bearing, including those made with heavy aggregates is 500 psi. These low strength requirements have restricted the use of precast concrete masonry units in Britain. On the other hand, in the United States, strength requirements are comparatively high, but high strengths by ordinary curing, *i.e.*, other than high pressure curing, give high shrinkages.

A careful analysis of shrinkage cracking in concrete masonry walls has been made, but just what portion of masonry wall cracks is caused by shrinkage is indeterminate. It is clear that shrinkage does cause many cracks but it is not within the scope of this paper to set down shrinkage values which will guarantee against shrinkage cracks in walls.

Much literature written between 1910 and 1930 dealt with high-pressure steam curing, mainly with sand-lime brick, mortars and the effect of autoclaving on cement. The first test applied to masonry units was reported in 1930.[1] The author, P. M. Woodworth, studied early strength, ultimate strength, strength as affected by the time interval between molding and curing, and shrinkage. He concluded that high-pressure steam cured units developed strengths in two days which were about 55 percent higher than the 28-day strength of moist-cured units, and were more than double the 2-day strength of units cured in low-pressure steam. He found a slight inexplicable retrogression in strength between the second and ninth day which then started to increase and continued beyond the 28th day. He found that the age at which the unit was subjected to high-pressure steam varied the strength only slightly. He concluded that high-pressure steam curing greatly reduced shrinkage of concrete masonry panels laid up with wet units; when air dry units were used he found practically no shrinkage.

W. D. M. Allen[2] agreed with Woodworth that high-pressure units shrink much less than others. It is interesting to note that he posed a problem 20 years ago which is still unsolved, "An equally important investigation which was not possible in this limited study is to determine to what extent a wall may shrink before cracking occurs."

J. C. Pearson and E. M. Brickett made a broad study[3] of varying temperatures and curing times. Several different types of cements were used. All their samples, however, were made with plastic concrete as contrasted to the dry nonplastic mixes used in concrete masonry. They showed that high-pressure steam curing gives higher strength, lighter color, greater resistance to volume changes, minimum crazing and cracking, control of porosity, better nailability and economy in cement. They also pointed out several disadvantages, *i.e.*, higher curing cost, some restrictions as to aggregate used, and high cost of equipment.* .

C. A. Menzel showed[6] that for high-pressure steam to be effective, a large amount of silica must be present in fine particles. This silica combines with the calcium hydroxide (lime), a product of the hydration of the cement, to form calcium silicates. These calcium silicates are cementitious which accounts for their comparatively high strength. The series of compounds formed by calcium silicates in the presence of aluminates form compounds called hydrogarnets which are crystalline in structure rather than amorphous and are less prone to expand and contract with the increase and decrease of water content. Leaching and efflorescence were shown to be greatly reduced by the use of high-pressure steam. Salts of this type do not form when a proper percentage of silica is present.

Menzel continued with his tests making them relate more to the masonry field.[7] He found silica content less important in the lean mixes characteristic of concrete masonry units than he had found it to be in his former investigations. He further concluded that the heating period from room temperature to 350 F and also the corresponding cooling period should be a minimum of 5 hours. As a result of his investigations, he also assigned to high-pressure steam cured block superior qualities with regard to shrinkage, cracking and spalling.

In Menzel's concluding test[8] he made a detailed study of proper age when steaming should start, proper time required to attain desired temperature, minimum requirements for steaming to avoid cracks and checks, aggregate-strength relationships and comparative bond strengths.

The most recent ACI article concerning high-pressure steam curing is the report of Committee 716.[15] The committee tried to define high-pressure steam curing in terms of linear shrinkage and the following tentative definition was proposed:

> High-pressure steam curing as applied to concrete products shall be understood to mean any process of curing in saturated steam under a pressure and for a period sufficient

*This last dieadvantage has been pointed out in other papers. This author believes, however, that the spread in cost between autoclaves with the accessory equipment and the equipment necessary to cure properly by the best atmospheric method is much smaller than is generally supposed.

to produce a stabilized finished product, such that a linear shrinkage thereof from a saturated condition to that after drying to constant weight at 212 F shall not exceed 0.03 percent.*

The committee apparently felt that if the shrinkage requirement could be met, regardless of the type of curing, the block should be deemed acceptable. It recognized the possibility of using a sufficiently low cement content to meet the shrinkage limitation, but felt that such a product would fail to meet strength requirements.

A private report in 1949[22] showed that within the workable range of moisture (5 to 40 percent of total moisture) high-pressure steam cured blocks shrank only 1/6 as much as those not so cured. It is conceded that differences in mixes may have been partly responsible for this great variation. This same report stated that the tests did not disclose any correlation between shrinkage under normal use and accelerated oven drying tests.

Only sketchy and inconclusive tests have been made comparing high-pressure steam cured units with those cured at atmospheric pressure†. Controlled low humidities were never used and units were not made by the same machine with the same batch. No tests were available in which blocks originally high-temperature steam cured were then high-pressured steam cured to get a comparison. A primary purpose of this study was to bridge this gap.

OBJECTIVES OF TESTS

The objectives of the test described were:

1. To run a controlled shrinkage test from one mixture of lightweight aggregate, cement and water making this typical of mixes being used for lightweight aggregate block in the United States; and further to cure samples of this mix by (a) the method considered best using atmospheric pressures,‡ and (b) the method generally used for high-pressure steam curing.

2. To use the British method (Appendix 1) for determining shrinkage and moisture movement of the products of each curing procedure, and to determine whether it could be used as a basis for an American specification.

MANUFACTURE AND CURING OF BLOCKS

The test blocks were made at 10:40 a.m. July 17, 1950 at the plant of Concrete Pipe & Products Co., Inc., Richmond, Va.§ They were made in a

*Actual tests on units apparently were not made in arriving at this limit. In practice, drying at 212 F with no specified humidity produces erratic shrinkage results because of chemical changes which take place. These changes seem to continue and to affect the shrinkage after the blocks reach a constant weight.

†It is generally conceded that a reputable manufacturer can produce blocks which will pass strength specifications regardless of the curing method. On the other hand, greater strengths can be obtained with a given amount of cement using high-pressure steam curing. This was shown in a private test report available from the author on request. The test compared strengths and shrinkages of air cured, high-temperature cured, and high-pressure steam cured units from the same batch. Shrinkage comparisons from saturation to "air dry" were also made which support this test.

‡Curing cycle suggested by National Concrete Masonry Assn. on request of Experiment, Inc. The intent was to use the method giving optimum curing using atmospheric pressure.

§Experiment, Inc., Richmond, Va., conducted the high-temperature curing and all test measurements while the manufacture of the blocks and the high-pressure steam curing were merely observed by their engineer. The test was sponsored by Arkhola Sand & Gravel Co., Fort Smith, Ark., Concrete Pipe & Products Co., Inc., Richmond, Va., Harter Marblecrete Stone Corp., Oklahoma City, Okla., National Brick Corp., Long Island City, N. Y., Pre-Shrunk Masonry Sales Corp., Roanoke, Va. and Roanoke-Webster Brick Co., Roanoke, Va.

TABLE 1—CURING CYCLES

Curing method	Holding, air at 70-90 F 90-95 percent R.H., hours	Heating, steam on	Constant temp., steam on	Soaking, steam off	Cooling time, 70-90 F, hours	Intermediate storage under tarpaulin, days	Final storage in air at 70 F, days
High-temp. steam	2	3 hr to 170 F	1 hr at 170 F	12 hr 170-125 F	5	5	21
High-pressure steam	2	3 hr to 350 F	8 hr at 350 F*	0	1	5	21

*An average temperature of 357 F was actually used.

Besser Super Vibrapac Model V3 M-136 conventional three-at-a-time block machine which molds under vibration and pressure. The mix was 376 lb Lehigh Type 1-A portland cement, 36 cu ft cinders, 70 lb low-carbon fly ash and enough water to make a typical block mix. This batch made 76 blocks. The machine was cleared before the batch was introduced so that the correct material would be used and so that the exact yield could be determined. The blocks were made on two consecutive pallets (three blocks per pallet) and were carefully marked. Three blocks (one pallet) were high-pressure steam cured in a 6 ft 4 in. diameter by 60 ft long autoclave. The other three blocks were cured by Experiment, Inc. The details of each curing cycle are given in Table 1.

Immediately after curing, all blocks were stored at Experiment, Inc., for the prescribed maturing. They were covered with a tarpaulin or waterproof paper during intermediate storage to prevent rapid drying, thus simulating normal slow drying in large stockpiles.

TEST SPECIMENS

After the blocks were fully matured, two test specimens each 6 x 3 x 1½ in. were cut with a masonry saw from each of the six parent blocks as shown in Fig. 1. The twelve specimens were divided into two test groups, A and B, of six specimens each, three specimens from each curing method in each test group. A third test group, C, of 6 specimens, composed of three high-temperature cured specimens from groups A and B were recured—three by high pressure and three by high temperature.

DISCUSSION

High-pressure steam cured blocks in every case had an average shrinkage about 50 percent less than blocks cured under atmospheric conditions (Table 2). This confirms previous tests using high-pressure steam curing. Earlier tests have shown that low-pressure steam and air cured blocks shrink even more than those cured by high-temperature steam.

In these tests, all the units cured by high-pressure steam passed the British test for shrinkage and moisture movement (Table 3). None of the high-temperature cured units passed either test.

TABLE 2—COMPARATIVE DRYING SHRINKAGE OF TEST BLOCKS*

Specimen No.	Curing method	Wet length, in.	Dry length, in.	Difference, wet and dry length, in.	Percent of dry length	Average, percent	Shrinkage, in. per 100 ft
			Test Group A				
1P	High-pressure steam	6.2932	6.2916	0.0016	0.025	0.0226	0.27
2P		6.3958	6.3946	0.0012	0.019		
3P		6.3240	6.3225	0.0015	0.024		
4T	High-temp. steam	6.4302	6.4265	0.0037	0.058	0.056	0.667
5T		6.4652	6.4618	0.0034	0.052		
6T		6.4125	6.4088	0.0037	0.058		
			Test Group B				
7P	High-pressure steam	6.3518	6.3499	0.0019	0.030	0.030	0.36
8P		6.2369	6.2351	0.0018	0.029		
9P		6.3281	6.3261	0.0020	0.032		
10T	High-pressure steam	6.3295	6.3269	0.0026	0.041	0.043	0.515
11T		6.3855	6.3826	0.0029	0.045		
12T		6.4939	6.4909	0.0030	0.046		
			Test Group C				
4TP	Recured high-pressure steam	6.4298	6.4280	0.0018	0.028	0.027	0.32
5TP		6.4670	6.4651	0.0019	0.029		
6TP		6.4140	6.4125	0.0015	0.023		
10TT	Recured high-temp. steam	6.3275	6.3249	0.0026	0.041	0.047	0.56
11TT		6.3839	6.3805	0.0034	0.053		
12TT		6.4920	6.4888	0.0032	0.049		

*Bulk density = 1.48

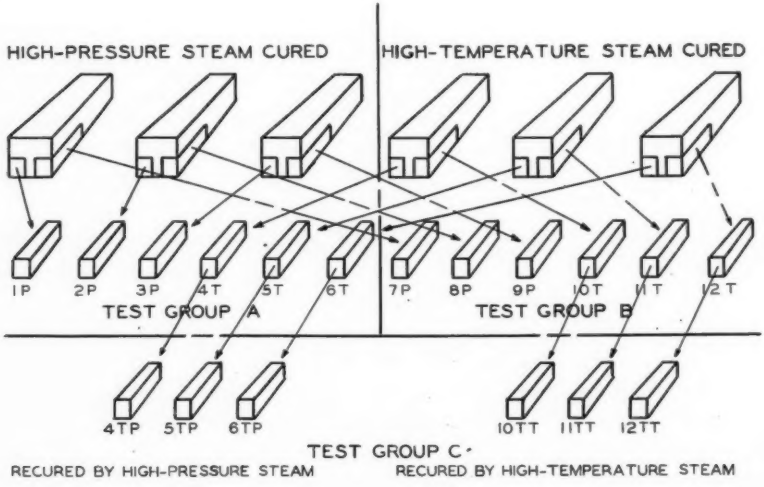


Fig. 1—Method of sample selection, curing and tests. All specimens tested for drying shrinkage and moisture movement

TABLE 3—COMPARATIVE MOISTURE MOVEMENT* IN TEST BLOCKS

Specimen No.	Curing method	Wet length, in.	Dry length, in.	Difference, wet and dry length, in.	Percent of dry length	Average, percent	Moisture movement, in. per 100 ft
			Test Group A				
1P	High-	6.2930	6.2916	0.0014	0.022		
2P	pressure	6.3958	6.3946	0.0012	0.019	0.0195	
3P	steam	6.3236	6.3225	0.0011	0.017		0.235
4T	High-	6.4294	6.4265	0.0029	0.045		
5T	temp.	6.4639	6.4618	0.0021	0.032	0.038	
6T	steam	6.4112	6.4088	0.0024	0.037		0.455
			Test Group B				
7P	High-	6.3511	6.3499	0.0012	0.019		
8P	pressure	6.2369	6.2351	0.0018	0.029	0.025	
9P	steam	6.3277	6.3261	0.0016	0.025		0.30
10T	High-	6.3294	6.3269	0.0025	0.040		
11T	temp.	6.3852	6.3826	0.0026	0.041	0.041	
12T	steam	6.4936	6.4909	0.0027	0.0415		0.49
			Test Group C				
4TP	Recured	6.4298	6.4280	0.0018	0.028		
5TP	high-	6.4669	6.4651	0.0018	0.028	0.027	
6TP	pressure steam	6.4141	6.4125	0.0016	0.025		0.324
10TT	Recured	6.3274	6.3249	0.0025	0.040		
11TT	high-	6.3838	6.3805	0.0033	0.052	0.047	
12TT	temp. steam	6.4920	6.4888	0.0032	0.049		0.565

*Measured expansion which occurs when a block is saturated after being dried to a stable condition at 122 F and 17 percent relative humidity.
Bulk density = 1.48.

Recuring of specimens 4T, 5T and 6T from group A (originally in the high-temperature group) by high-pressure steam, reduced drying shrinkage to approximately one-half its original value as shown by specimens 4TP, 5TP, and 6TP of group C. Recuring of specimens 10T, 11T and 12T from group B by high-temperature steam did not reduce drying shrinkage, but increased it slightly as shown by 10TP, 11TP and 12TP of group C. Moisture movement changes followed the same pattern with slightly smaller divergence (Table 3).

Although a limited number of samples were used, the test was run under controlled conditions and the results can be said to be indicative of what could be expected in more general tests. However, further tests broader in scope than this should be initiated and studies to determine shrinkage with different aggregates should be begun as soon as possible.

CONCLUSIONS

1. High-pressure steam cured precast masonry units had less drying shrinkage than high-temperature steam cured units.

2. Moisture movement in high-pressure steam cured masonry building units was less than in high-temperature steam cured units.

3. Drying shrinkage and moisture movement in high-temperature steam cured units can be reduced by recuring with high-pressure steam.

4. After minimum curing time with high-temperature steam, continuation of curing does not reduce drying shrinkage or moisture movement. Continued high-pressure steam curing of blocks originally high-temperature steamed will reduce drying shrinkage and moisture movement. This indicates that the 170 F maximum temperature in the high-temperature method does not carry the potential chemical changes to completion, whereas the 357 F maximum temperature and moisture conditions in high-pressure curing does allow this chemical change to progress further.

5. The British standard method of measuring drying shrinkage and moisture movement is a practical, relatively quick method of testing drying shrinkage, and might be used as a basis for writing a specification for masonry units manufactured in this country. A re-examination of present specifications is evidently desirable since concrete masonry units which shrink rapidly because of inadequate specifications or inadequate curing discourage their use.

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APPENDIX 1

TEST PROCEDURE

Method for the determination of drying shrinkage

The following methods for determining drying shrinkage and moisture movement are quoted from the British Standards Institution booklet, "Precast Concrete Blocks, Standards 492, 728 and 834," 1944.

A section, not less than 6 in. long and as near as is practicable 3 x 3 in. in cross section, shall be cut from each block and prepared for measurement. The sections shall be completely immersed in water for four days, the temperature being maintained at 20 C ± 1 C (68 F ± 2 F) for the final four hours. Immediately after removal of the specimens from the water the length of each shall be measured and these values shall be taken as the wet lengths.

The specimens shall then be dried to constant length in an air-oven maintained at 50 C ± 1 C (122 F ± 2 F) in which the humidity is controlled at 17 percent relative humidity by means of saturated calcium chloride solution. The specimens shall be removed from the oven, cooled for two hours in a desiccator containing anhydrous or a fully-saturated solution of calcium chloride, and the length of each then measured. When constant length is attained, *i.e.*, when the difference between two consecutive readings separated by a time interval of 48 hours is not more than that equivalent to ± 0.0002 in. on a 6 in. length, this shall be taken as the dry length.

The shrinkage shall be determined as the difference between the wet and dry lengths and expressed as a percentage of the dry lengths.

Particulars regarding oven—Not more than six specimens shall be placed in an oven of 1.8 cu ft capacity. If a larger oven is utilized the maximum ratio of the number of specimens to cubic capacity must be the same.

The wet specimens shall be placed in the oven at the same time, *i.e.*, wet specimens shall not be placed in an oven which already contains partially dried specimens.

The oven shall be moderately air-tight and shall be fitted with a fan to keep the air circulating efficiently in the oven.

Humidifier—In order to ensure saturation of the calcium chloride solution, solid calcium chloride shall always be present throughout the test.

When six specimens are employed there shall be in the oven a minimum of 1000 ml saturated calcium chloride solution, contained in crystallization dishes or trays such that there is approximately 1 sq ft of surface exposed. If a larger oven is used to deal with more specimens, then for each specimen above six an extra 150 ml of solution must be added.

The calcium chloride solution shall be thoroughly stirred at least once every 24 hours.

Method for the determination of bulk density

The dried sample used for the determination of drying shrinkage shall, after removal of the steel balls from the ends, be used for making the determination of the bulk density as follows:

The dimensions of the dried sample shall be measured in cm and the bulk volume in ml computed. The specimen shall then be weighed in grams and the bulk density calculated from the ratio:

$$\text{Bulk density} = \frac{\text{weight of specimen in grams}}{\text{volume of specimen in ml}}$$

Method for the determination of moisture movement

The specimens, which have previously been used for the drying shrinkage test shall, after the completion of that test, be immersed in water for four days, the temperature being maintained at 20 C ± 1 C (68 F ± 2 F) for four hours previous to the removal of the specimens and the length measured. The moisture movement shall be determined as the difference between the dry and wet lengths and expressed as a percentage of the dry length.

Drying shrinkage

The drying shrinkage of the blocks when unrestrained shall be determined by measurements of three specimens, cut one from each of three blocks taken at random. The shrinkage of each specimen so measured shall not exceed the following values:

Blocks of bulk density (g per ml) over 1.4—0.04 percent
Blocks of bulk density (g per ml) 1.4 or less—0.06 percent

Moisture movement

The moisture movement of the dried blocks on immersion in water shall not exceed the following values:

Blocks of bulk density (g per ml) over 1.4—0.03 percent
Blocks of bulk density (g per ml) 1.4 or less—0.05 percent

Additional test conditions

Preparation for measurement comprised cementing a ¼-in. diameter steel ball in the center of each end of each specimen with half of the surface of the ball projecting. After one day in moist air to allow the cement to harden the specimens were completely immersed in water for four days.

The test specimens prepared for this test were 6 x 3 x approx. 1½ in. It was decided to cut the specimens from a standard hollow block, rather than make up a special solid block, thus insuring that the test specimen though thinner in one dimension would have the same joggling, molding pressure and bulk density as a standard block.

These procedures were followed for each test group.

Discussion of a paper by Harry W. Easterly, Jr.:

Correlation of Shrinkage and Curing in Concrete Masonry Units*

By ALBYN MACKINTOSH, M. SPINDEL and AUTHOR

By ALBYN MACKINTOSH†

The author is to be complimented for presenting additional and useful information on the problem of shrinkage in concrete masonry structures. It seems quite evident that it should now be generally recognized that high temperature steam curing does not answer the problem of shrinkage. High pressure steam curing has many good points but its economic justification is not altogether clear.

Where the same manufacturer's block may be used in areas where relative humidity reaches 3 percent, or in an area where relative humidity reaches 20 percent or higher, and in some cases used under saturated conditions and where temperatures likewise differ tremendously, the British standard set forth in this article does not seem to be adequate to give the answers required.

In trying to find an answer to the problem of shrinkage in this area we have had the cooperation of members of the Concrete Masonry Manufacturers Assn. and to date have made several hundred tests. These tests have generally consisted of taking a cured block and measuring the shrinkage from the saturated to the oven-dried condition, then attempting to correlate these results with observed performance of similar units in concrete masonry structures. Although there is some indication that this simple shrinkage test does not give the actual linear shrinkage developed in a period of time, there is considerable evidence that there is good correlation between the simple test and field performance in determining what units are likely to be involved in shrinkage problems. We expect to have enough data collected in another year of this program to yield some definite results. At the present time we are recommending the following specification for hollow concrete masonry construction and are obtaining good results therefrom.

> Masonry units shall be Grade A units conforming to the "Standard Specifications for Hollow Load-Bearing Concrete Masonry Units," ASTM Designation C 90-44.
>
> Masonry units shall be not less than 28 days old, shall have had a two weeks' drying cycle prior to their use in the structure, and shall have a maximum linear shrinkage of 0.08 percent from the saturated to the oven-dry condition.

*ACI Journal, Jan. 1952, *Proc.* V. 48, p. 393. Disc. 48-27 is a part of copyrighted Journal of the American Concrete Institute, V. 24, No. 4, Dec. 1952, Part 2, *Proceedings* V. 48.

†Consulting Engineer, Los Angeles, Calif.

At the time of laying masonry units, their moisture content shall be as near as possible the same as the ultimate moisture content expected in the structure. The units shall not be wet before laying and if rain occurs prior to laying, they shall be dried before use.

Where a more rigid specification is desired, it is recommended that the linear shrinkage be changed to read "shall have a maximum linear shrinkage of 0.06 percent from the saturated to the oven dry condition."

By M. SPINDEL*

The author is right in stating that "the present specification which limits the amount of moisture in the unit when it is laid is inadequate" for limiting the amount of shrinkage and its detrimental consequences. Shrinkage of mortar and concrete during setting, hardening and drying depends not only on the percentage of water at the beginning of the test, but on the type and properties of raw materials used and their proportions; the method of making, curing, special treatment, drying and storing; the texture of mortar and concrete before and after setting and hardening, especially with regard to the percentage of voids and the size, shape and distribution of voids and capillary pores filled with water and/or air. The percentage of shrinkage determined by tests depends also on the method of making these tests in the laboratory and in the field.

Shrinkage of conventional heavy concrete has been investigated over a number of years and many ways have been used successfully to avoid or reduce the damage resulting from shrinkage, especially to structures such as dams, pavements, bridges, viaducts, etc. The question of avoiding shrinkage cracks in concrete houses took on added importance when the manifold new types of lightweight concrete came into use.

Certain natural or even artificial lightweight raw materials known to have the highest insulating capacity if completely dry, absorb abnormally large amounts of water and thus lose their valuable properties. In making lightweight concrete of such raw materials the water comes at first from the mixing water which often is far too much and has no easy way of escape. If such concrete is not damp-proofed and/or waterproofed it is, of course, permeable to dampness and water from soil and rain.

In discussion of a paper by Bruce E. Foster, "Use of Admixtures in Concrete Products,"† R. E. Madison‡ said that according to his experience mortar joints are considerably less subject to passage of water than are block and that a great number of block examined were found so open-textured that water could almost be poured through them. The author of the forementioned paper agreed that many concrete block offer little resistance to the passage of water and their surface must therefore be parged or otherwise treated to prevent water passage.

It has been noted that at first it was considered an "advantage" to sacrifice strength, resistance to wear and durability to obtain light weight and high

*Research Engineer and Consultant, London, England.
†Foster, Bruce E., "Use of Admixtures in Concrete Products," ACI JOURNAL, Sept. 1950, *Proc.* V. 47, p. 32.
‡ACI JOURNAL, Part 2, Dec. 1951, *Proc.* V. 47, p. 52-2.

insulation, but often the latter, too, are lost if the problems of lightweight concrete are not dealt with on the same sound basis of theoretical and practical knowledge as those of heavy concrete. In both cases we have to consider the percentages in absolute volumes of cement, aggregates, water and air and the influence of water-cement ratio and void-cement ratio both in fresh and hardened concrete. We also have to understand the meaning and testing of water-absorption and water-saturation coefficients and to make use of the special advantages obtained with admixtures and special treatments.

Regarding the quality improvement of concrete in general, and of lightweight concrete especially, there is general agreement that high pressure steam curing has a highly beneficial effect on most properties apart from reduced drying shrinkage and moisture movement. This beneficial effect is due partly to some improvements of the hardened cement *per se* and partly to chemical reaction of the lime of the hardened cement with the silica of the sand particles.

The tests made by the author on high pressure steam cured units made of portland cement and cinders with an addition of fly ash are of great interest. More information on the composition of this lightweight concrete, especially the percentages of water and air, bulk density and strengths will be much appreciated, as well as results of tests where strengths and shrinkage of units cured in different ways were compared.

Of course, the purpose of the tests published in the paper was to deal with the most suitable method of testing drying shrinkage and moisture movement and to standardize limits for both. There may be different ways of making such tests. At present nearly all of them do not yet consider or measure the shrinkage of mortar and concrete which occurs from the very beginning. As the writer pointed out in various papers,* shrinkage of mortar and concrete begins as soon as the water is mixed with cement and can easily be determined by volume. Diagrams in the forementioned paper show that the amount of shrinkage during setting and soon afterwards may be larger than after many days, weeks or months. This neglected part of shrinkage, which is mostly irreversible, is often of great importance for mortar and concrete placed in situ.

There are, of course, various views not only with regard to the method of testing but also with regard to the limits to be allowed for drying shrinkage. These vary from 0.03 percent† to 0.06 percent for concrete giving little shrinkage trouble, between 0.06 and 0.10 percent for concrete of a fair performance according to the Housing and Home Finance Agency,‡ and the limits according to British Standard Specifications 492, 728 and 834 (1944) given in Appendix 1 of the paper. Of these specifications, B.S. 492 and 728 (1944) for solid and hollow partition block, respectively, limit drying shrinkage to 0.08 percent for block of this type and B.S. 834 (1944) for block for rendered walls limits drying shrinkage to 0.04 percent for block of bulk density over

*Spindel, M., Question *iii*, Special cements, Second Congress on Large Dams, Washington, D. C., 1936.
†ACI Committee 716, "High Pressure Steam Curing," ACI JOURNAL, Apr. 1944, *Proc.* V. 40, pp. 409-416.
‡"Lightweight Aggregate Concrete," Housing and Home Finance Agency, Washington, D. C., 1950.

1.4 gm per ml and 0.06 percent for block of bulk density below 1.4 gm per ml. Whitaker* points out that these limits "do not apply to aerated concrete" and gives a warning that "the use of a building block with drying shrinkage and wetting expansion within the limits of these specifications will not automatically ensure a crack-free structure; design and workmanship must contribute their quota."

It was a wise suggestion by the author that British standard methods "might be used as a basis for writing a new specification." Of course, it will have to be considered that every standard specification, even at the time of issuing, is only a compromise between what is needed and what can be accomplished and continuously has to be improved according to experience. Recent experience would have to be taken into account before deciding to make use in 1952 of standards written in 1944 and based on the experience before that date.

AUTHOR'S CLOSURE

It is indeed gratifying to note that the discussers agree with the author that the measurement of relative absorption is not the proper method to measure shrinkage in block.

Even where conditions more severe than 17 percent relative humidity at 122 F occur, the value of the British test as a measure of shrinkage under advanced conditions is not lost. It still remains a positive measurement. The author feels that Mr. Mackintosh puts too much faith in the test which measures shrinkage from a saturated to an oven-dry condition. This test gives erratic results because of subtle chemical changes which take place with low moisture conditions. Furthermore, in the specification suggested by Mr. Mackintosh and the California group, high pressure steam curing was unnecessarily penalized by the 28-day age limitation. This is quite often the case even in areas where high pressure steam cured block are available. In an effort to keep the specifications open to block using other types of curing, stringent conditions are laid down which strongly penalize the high pressure steam manufacturer when this product is admittedly superior.

It is further felt that Mr. Mackintosh's minimum specification of 0.06 percent will still give shrinkage cracks. None of the tests on high temperature steam cured block in the paper under discussion showed results this high. It is admitted that block cured by the high temperature method give shrinkage cracks.

Mr. Spindel touches on an important point when he mentions the high permeability of block. A block with a high permeability might hold 5 lb of water in a saturated condition; one with a low permeability might hold only 3 lb. It is not consistent to say that the former will pass the specification (40 percent limit) if it contains 2 lb of water while the latter will fail if it contains more than 1.2 lb. This illustrates the inadequacy of the 40 percent limitation rather than an actual shrinkage determination.

*Whitaker, T., "Lightweight Concretes: A Review," Building Research Congress, 1951.

Mr. Spindel aptly points out that shrinkage limitations go from 0.03 to 0.10 percent or higher. More information is needed to determine a shrinkage percentage which will provide safety from wall cracking. The author does not believe the British specification to be the final word in block shrinkage determinations but does endorse some of its principles, namely:

(a) that a measurement of shrinkage itself rather than some other property should be made on the sample to be tested;
(b) that the measurements should be made at saturation and at some specified temperature and humidity;
(c) that the test should be relatively easy to run;
(d) that the test should be inexpensive;
(e) that the test should be run in a short time.

It is possible that samples should be full-size block and that humidity and temperature should be more extreme. This will be a simple matter to determine. At the present time ACI Committee 716 is conducting tests which may show an improvement on the British method.

Mr. Spindel has asked for the report comparing shrinkages and strengths of block made by different curings. The following is a summary of this test, with results given in Table A.

The mixture used was one bag of cement, 504 lb of damp cinders, 17 lb of fly ash. The air cured block, low pressure steam cured block and high pressure steam cured block were all made from the same batch, and on the same block machine.

Air cured block—These block were given the usual air curing, namely they were kept moist for 24 hr., when they were placed outside and were left there until tested.

Low pressure steam cured block—These block were allowed to set for 2 hr. These block were given steam, raised at the rate of 40 F per hour until 180 F was reached and held there for 12 hr. The steam was turned off and block allowed to soak for 6 hr. These block were then placed in a dryer maintained at 180 F for 5 hr. They were then given the same treatment as the air cured block.

High pressure steam cured (autoclaved) block—These block were placed in an autoclave and allowed to set for 2 hr. The steam was turned on and the pressure raised to an average of 125 psi (temperature 350 F) and maintained

TABLE A—COMPARISON OF SHRINKAGE AND STRENGTH OF BLOCK CURED BY DIFFERENT METHODS

	Air cured	Low pressure steam cured	High pressure steam cured (autoclaved)
Moisture content at time of delivery (percent of total absorption)	29.47	30.66	11.32
Compressive strength at 28 days (psi)	831	1010	1212
Shrinkage from saturation to air dry (in. per 100 lin. ft)	0.375	0.357	0.189

for 8 hr. The steam was released and the block remained in the autoclave for 8 hr. The block were then removed from the autoclave and stored outside with the others until tested.

Shrinkages in Table A were plotted at convenient intervals. The shrinkage at "air dry" (20 percent of moisture present in saturated condition) was the value noted. Erratic results were evident when moisture was reduced any considerable degree below this point. It should be noted that shrinkages for air cured block and low pressure steam cured block were approximately the same and that high pressure steam cured block showed shrinkages of about half these values. Strength results show great advantages for high pressure steam curing.

Shrinkage cracks may never be eliminated until manufacturers uniformly cure by high pressure steam. No alternate satisfactory solution has yet been offered. Although equipment for this type of curing is expensive, it is not so far out of reach of the average producer as is generally supposed. It offers many solutions to the problems of block making and its general use would greatly upgrade the concrete products industry.

Title No. 48-28

Equivalent Load Method for Analyzing Prestressed Concrete Structures*

By ROBERT B. B. MOORMAN†

SYNOPSIS

A method is presented for post-tensioned reinforced concrete structures whereby the effect of cable tension can be expressed as a distributed or concentrated load. The analysis thus becomes a simple matter of applying methods with which the designer is already familiar. The function of the post-tensioned wires thus can be visualized easily.

INTRODUCTION

The effect of post-tensioned wires can be simply expressed in terms of a given loading condition on a beam or structure. A structure of this type is in a sense similar to a self-anchored suspension bridge or possibly a trussed beam. For instance, assume a simple beam of constant section as in Fig. 1 to have a parabola or circular-arc for the shape of the post-tensioned wires. The effect of this post-tensioning on the beam is the same as a uniformly distributed load acting upward plus a direct compression. If the post-tensioned wires at the ends are located at the neutral axis (or center of gravity) there is no moment due to eccentricity at the ends. The post-tensioned load then could be made to actually counteract a uniform dead load. Consequently, only direct compression would exist in the member.

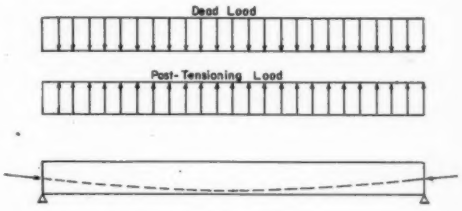


Fig. 1—Location of prestressing cables for uniformly distributed load on simple beam of constant cross section

A method of fully counteracting a concentrated load is shown in Fig. 2. The post-tensioned wires at the ends should be placed at the centroid of the end section of the beam. Along the beam the wires should form two straight segments. In effect this is essentially a trussed beam.

*Presented at the ACI St. Louis Regional Meeting, St. Louis, Mo., Oct. 31, 1951. Title No. 48-28 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 23, No. 5, Jan. 1952, *Proceedings* V. 48. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than May 1, 1952. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Professor of Civil Engineering, University of Missouri, Columbia, Mo.

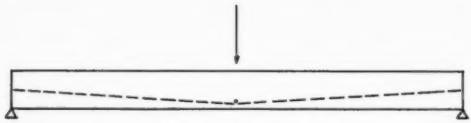


Fig. 2—Location of prestressing wires for concentrated load on simple beam



Fig. 3—Method of counteracting two concentrated loads on a beam

A manner of counteracting two concentrated loads is shown in Fig. 3. The post-tensioned wires are straight between reactions and loads. Actually the wires may be considered as the rays of a string polygon.

In the case of a continuous beam we should properly place the post-tensioned wires as shown in Fig. 4. These wires then induce the same effect on the beam as a uniformly distributed upward load. Thus a uniformly distributed gravity load can be counteracted in this manner.

Take the case of a fixed-ended beam with a uniform load. This can be worked out as shown in Fig. 5. Here the reinforcement is disposed in such manner as to counteract the distributed load. If the supports are unyielding, there will be no direct thrust in the beam.

Various cases are discussed in detail to demonstrate the principles involved in the use of the equivalent load method. Then, some rules are presented for use in the general case.

NOTATION

Letter symbols are defined where they first appear, in text or illustrations, and are assembled in Appendix 1 for ready reference.

DERIVATIONS

In the derivations which follow, the friction between the prestress cable

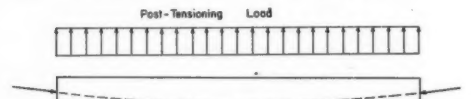


Fig. 4—Prestressing wires in continuous beam

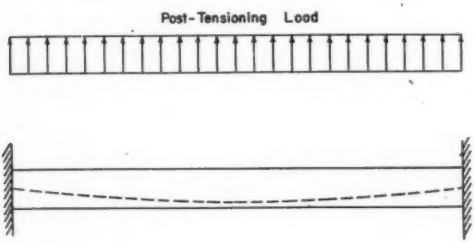


Fig. 5—Prestressing wire pattern in fixed-end beam

and the concrete is neglected; however, it can be included, as well as shrinkage and expansion. It is assumed that the members are elastic and that deflections of the structure do not alter its dimensions for purposes of analysis. In the first part of the paper only beams symmetrical about their gravity axes are considered. This in no way should be taken as a limitation of the method here presented for it applies to irregular members as well.

The distributed load resulting from post-tensioning may be expressed in terms of the post-stress and the radius of curvature of the reinforcement. Suppose the reinforcement lies on a parabola or flat circle. Now, since the wire is not bonded to the concrete but is actually post-tensioned, the tension in the wires may be expressed as

$$T = rw_p$$

and, solving for w_p

$$w_p = \frac{T}{r}$$

in which T = tension in post-stressed wires
r = radius of curvature of wires
w_p = equivalent load resulting from post-tensioning

For a very flat curve the points on the circle and on the parabola lie along practically the same line.

Fig. 6 shows one-half of a beam with end connections of the post-tensioned wires at mid-depth and a center eccentricity e. For this simple beam the post-stressed wires are placed along the circular-arc as shown.

An expression can be derived for the radius of curvature, in terms of the span L and center eccentricity e, by use of the Pythagorian theorem. Thus,

$$r^2 = \left(\frac{L}{2}\right)^2 + (r - e)^2$$

from which

$$r = \frac{L^2}{8e} + \frac{e}{2} = \frac{L^2 + 4e^2}{8e}$$

The tension in the wires is expressed as

$$T = rw_p = \frac{L^2 + 4e^2}{8e} w_p$$

The assumption that the horizontal component is equal to the tension in the wires results in an error of less than two percent.

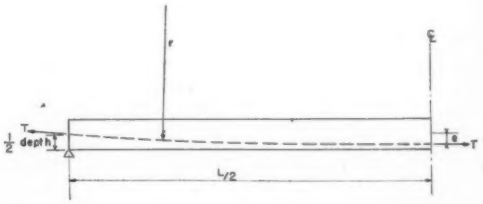


Fig. 6—Location of end connections and prestressing wire in mid-span of beam

Two-span continuous beam of constant section

The two-span continuous beam in Fig. 7 has the post-tensioned wires disposed along a parabola. The beam is of constant section and the end connections for the wires are at mid-depth. For this case the equivalent upward load may be expressed as

$$w_p = \frac{T}{r}$$

An expression for r may be developed as follows:

$$r^2 = L^2 + (r - e)^2$$

from which

$$r = \frac{L^2 + e^2}{2e}$$

Then

$$T = \frac{L^2 + e^2}{2e} w_p$$

or

$$w_p = \frac{2Te}{L^2 + e^2}$$

The beam in Fig. 8a is used to illustrate the method of handling the equivalent load concept. Assume the cables have been post-tensioned to a total stress T. Remove support 2 as in Fig. 8b. The cable tension, T, causes an upward load, w_p, which in turn produces an upward deflection as in Fig. 8c. Say this upward deflection can be computed as

$$\delta_2 = \frac{5\, w_p\, (2L)^4}{384\, EI}$$

in which E = modulus of elasticity
and I = moment of inertia of the cross section.

To reduce the deflection at support 2 to zero, there must be a reaction R_2 producing a downward deflection δ_2' such that

$$\delta_2' = \delta_2$$

The deflection shown in Fig. 8d may be expressed as

$$\delta_2' = \frac{R_2\, (2L)^3}{48\, EI}$$

It is now possible to solve for R_2 by use of the condition $\delta_2' = \delta_2$, or,

$$\frac{R_2\, (2L)^3}{48\, EI} = \frac{5 w_p\, (2L)^4}{384\, EI}$$

from which

$$R_2 = \frac{5 w_p L}{4}$$

Upon substitution for w_p, one obtains

$$R_2 = \frac{5TeL}{2(L^2 + e^2)}$$

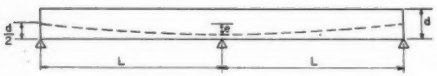


Fig. 7—Curved prestressing wires in two-span continuous beam of constant section

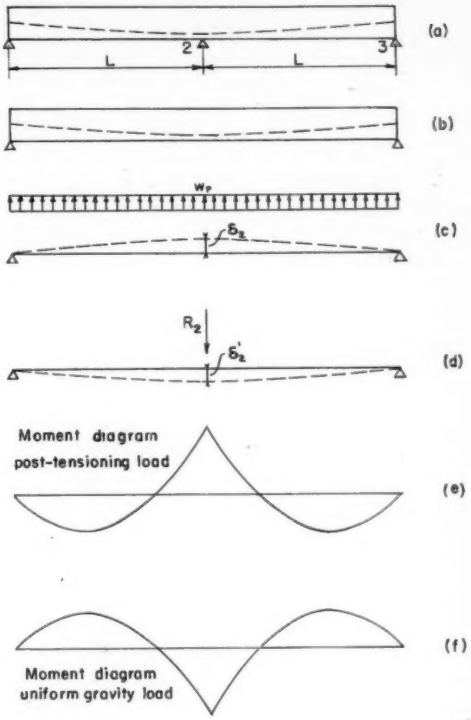


Fig. 8—Application of equivalent load method to continuous beam

The bending moment diagram resulting from the action of w_p is shown in Fig. 8e. The bending moment diagram caused by a uniform gravity load is shown in Fig. 8f. For the condition in which these two loads are equal the resulting bending moment diagram would be a straight line indicating zero moment. Throughout this discussion it must be kept in mind that there is a direct compression, P, in the beam equal to practically the full value of the tension T.

The variation in the moment of inertia as a result of the cable sheath being located at various heights along the beam has been neglected. It would complicate the procedure but slightly to include this effect.

It should be noted that the continuous beam, with load w_p, could have been analyzed by use of either the moment distribution method or the slope-deflection method.

Simple beam of constant section, axis parabolic

Consider the beam in Fig. 9. The post-tensioned wires are straight, and, located at mid-depth at the ends and distance e below the neutral axis at

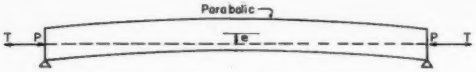


Fig. 9—Straight prestressing wires in parabolic beam

midspan. The depth is constant. Both top and bottom of the beam have slight parabolic curvatures, concave downward.

The moment curve, resulting from post-tensioning, has the same shape as for a uniformly loaded simple beam. The equivalent uniform load may be determined by using the condition that the bending moments are equal at midspan. Thus,

$$Te = \frac{w_p L^2}{8}$$

from which

$$w_p = \frac{8Te}{L^2} \text{ (upward)}$$

Simple beam of constant section, axis gabled

The beam in Fig. 10 has straight post-tensioned wires. It consists of two straight segments placed in such manner that a peak is formed at midspan. The end connections are at mid-depth and the eccentricity at midspan is distance e below the neutral axis.

In this case the equivalent loading is a concentrated load at midspan. Obviously, this load, P_p, acts upward. Because the post-tensioning moment has the same shape as for a concentrated load at midspan, the bending moments at this point are equated to determine the equivalent load. Thus,

$$Te = \frac{P_p L}{4}$$

from which

$$P_p = \frac{4Te}{L}$$

Simple beam of variable section, axis parabolic

Consider the beam in Fig. 11 which has straight post-tensioned wires. The top is straight and the bottom is of parabolic shape. The end connections are at mid-depth and the eccentricity of midspan is e distant below the neutral axis.

The equivalent load may be found easily in the following manner. The depth of the beam any distance x from midspan may be expressed as

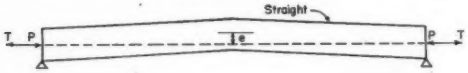


Fig. 10—Straight tensioning wires in gabled beam

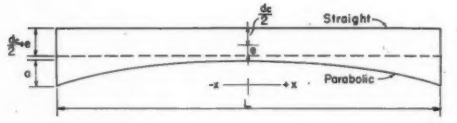


Fig. 11—Straight post-tensioning wires in beam with straight top and parabolic bottom

$$d_x = d_c + 4a\frac{x^2}{L^2}$$

in which d_x = depth of beam at any distance x from midspan
d_c = depth of beam at midspan
a = haunch (see Fig. 11)

The eccentricity at any distance x may be expressed as

$$e_x = \frac{d_c}{2} + e - \frac{d_x}{2} = \frac{d_c}{2} - e - \frac{d_c}{2} - \frac{4ax^2}{2L^2} = e - \frac{2ax^2}{L^2}$$

The bending moment, at any distance x, caused by the post-tensioned wires is

$$M_x = Te_x$$

If at the ends, the post-tensioned wires are at $d/2$ then $M_{L/2} = 0$ and the moment varies as a parabola. Hence the equivalent load is a uniform load. The load may be determined by equating the moments at midspan and solving for w_p, as

$$Te = \tfrac{1}{8}\, w_p\, L^2$$

from which

$$w_p = \frac{8Te}{L^2}$$

It is an easy matter to include the effect of eccentric end connections. A moment is applied at the end equal to the tension (actually horizontal component) times the eccentricity.

Continuous beam, constant section, straight wires

The two-span continuous beam in Fig. 12a is of constant section with straight post-tensioned wires. Post-tensioning in this manner is equivalent to applying a negative moment at each end of the beam. The structure may be analyzed in the following manner.

Remove support 2 and determine the displacement at 2 caused by post-tensioning the wires. The deflection, indicated in Fig. 12b, is

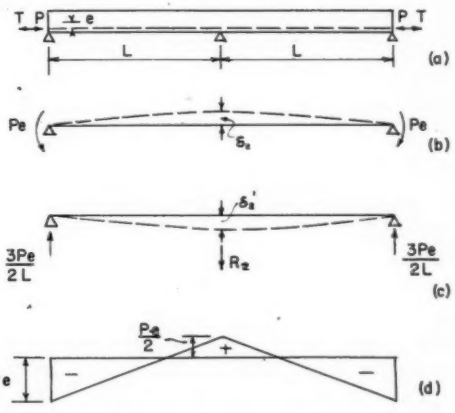


Fig. 12—Analysis of continuous beam with straight prestressing wires

$$\delta_2 = \frac{Te(2L)^2}{8EI} \quad \text{(upward)}$$

Now pull down at 2 with a reaction R_2, as shown in Fig. 12c. This displacement may be expressed as

$$\delta_2' = \frac{R_2(2L)^3}{48EI}$$

The condition of no displacement at 2 is used to obtain an expression for R_2. That is $\delta_2' = \delta_2$, or,

$$\frac{R_2(2L)^3}{48EI} = \frac{Te(2L)^2}{8EI}$$

from which

$$R_2 = \frac{3Te}{L}$$

The bending moment diagram as a result of the post-tensioning, T, is shown in Fig. 12d.

The structure could be analyzed easily by either moment distribution or slope-deflection. The loading condition is a negative bending moment $Pe = Te$ applied to each end of the beam.

Continuous beam, variable eccentricity

Consider the two-span continuous beam in Fig. 13a. The post-tensioned wires are straight with the eccentricities of the end connections equal to zero. The section of the beam is constant but its axis is in the form of two parabolas. The midspan eccentricity is indicated as e.

Fig. 13b shows the bending moment diagram resulting from the post-tensioning moment only, support 2 removed. Noting this moment diagram, the deflection at 2 for the simple span 1-3 may be expressed as

$$\delta_2 = \frac{\frac{2}{3}(Te)\,L}{EI} \cdot \frac{L}{2} \quad \text{(upward)}$$

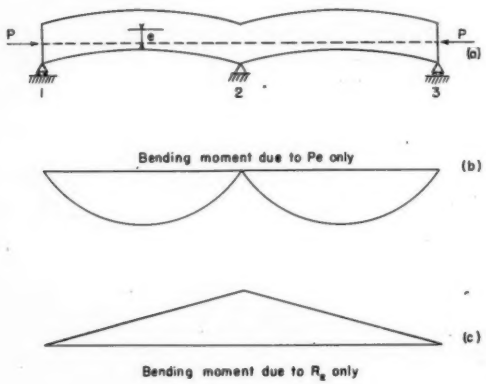


Fig. 13—Continuous beam with variable eccentricity

If a reaction R_2 is assumed to act downward at 2, a resulting bending moment will be as shown in Fig. 13c. Using the bending moment diagram, shown in the figure the deflection at 2 may be expressed as

$$\delta_2' = \frac{R_2 L^2}{4EI} \cdot \frac{2L}{3} \text{ (downward)}$$

The reaction at 2 may be determined from the condition

$$\delta_2' = \delta_2$$

or

$$\frac{R_2 L^3}{6EI} = \frac{TeL^2}{3EI}$$

whence

$$R_2 = \frac{2Te}{L} = \frac{w_p L}{4}$$

since $w_p = 8Te/L^2$ the bending moment at 2 is

$$M_2 = \frac{R_2 L}{2} = \frac{w_p L^2}{8}$$

It should be noted that the bending moment, M_2, is the same as for a two-span continuous beam subjected to a uniform load w_p. The reactions are the result only of the moment due to continuity. In other words, this structure may be analyzed in the usual way for moment by use of the equivalent load w_p. After the moment at support 2 is determined, equate the equivalent load to zero to determine the reactions and shear. A zero equivalent load for other than the bending moments results from the straight post-tensioning wires.

Continuous beam, variable section

Fig. 14a shows a two-span continuous beam. The end connections are at

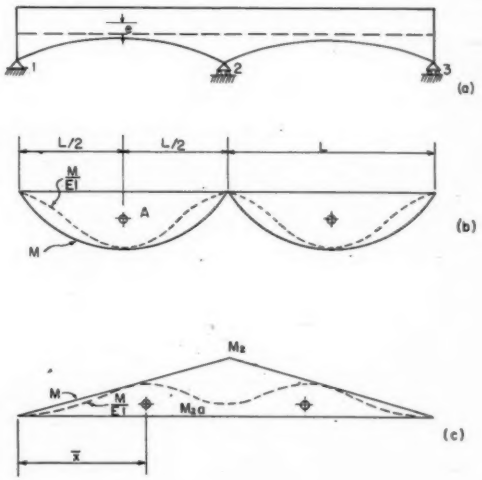


Fig. 14—Symmetrical continuous beam symmetrically loaded by post-tensioning

mid-depth and each midspan eccentricity is e. The upper side is straight. The lower side is curved parabolically. The post-tensioning wire is straight.

The structure is symmetrical and symmetrically loaded by post-tensioning. The tangents to the axis of the beam at 2 remain unchanged. It therefore follows that in deriving an expression for the moment at 2 only one span need be considered.

If A is the area of the M/EI diagram resulting from post-tensioning (see Fig. 14b) and, if span 1-2 is assumed to be simple, the change in slope at 2 is

$$\alpha_2' = \frac{A\frac{L}{2}}{L} = \frac{A}{2} = \int_0^L \frac{M'dx}{2EI} = \int_0^L \frac{w_p(Lx-x^2)dx}{4EI}$$

For a moment, M_2, applied at 2, if a_1 is the area of the M/EI diagram for M_2 = unity, and $\bar{x}$ is the distance from 1 to the centroid of this area (see Fig. 14c), the change in slope at 2 due to M_2 is

$$\alpha_2 = \frac{\bar{x}M_2a_1}{L} = \frac{M_2}{L}\int_0^L \frac{x^2dx}{EI}$$

The condition that there is no change in slope at 2 may be expressed as

$$\alpha_2 = \alpha_2'$$

or

$$\frac{M_2}{L}\int_0^L \frac{x^2dx}{EI} = \int_0^L \frac{w_p(Lx-x^2)dx}{4EI}$$

whence

$$M_2 = \frac{\int_0^L \frac{w_p(Lx-x^2)dx}{4EI}}{\int_0^L \frac{x^2dx}{LEI}}$$

The above expression gives the same moment at 2 as would be obtained by either slope-deflection or moment distribution. Since the post-tensioning wire is straight the equivalent load w_p would be used to determine the moment M_2 by any suitable method, then w_p would be set equal to zero to determine reactions and shears.

Equivalent load

The moment at any point due to post-tensioning may be expressed as

$$M = -Te$$

Then

$$\frac{dM}{dx} = -T\frac{de}{dx}$$

$$\frac{d^2M}{dx^2} = -T\frac{d^2e}{dx^2} = -w_p$$

When it is not possible to conveniently express the variation of e mathematically, the following finite difference expression may be used

$$\frac{d^2e}{dx^2} = \frac{e_1 - 2e_2 + e_3}{(\Delta x)^2} \text{ approximately}$$

in which e_1, e_2, e_3 = eccentricity at any three consecutive points along the beam and Δx is the distance between the points measured along the beam. The equivalent load can then be expressed as

$$w_p = T \frac{e_1 - 2e_2 + e_3}{(\Delta x)^2}$$

This expression may be used for members of variable section. The eccentricity e is positive when measured below the centroid of the section. A negative w_p indicates that the load is acting upward. The beam should be divided into equal length segments Δx.

If the post-tensioning wire is straight the equivalent load w_p is used only for determining the moments, after which, it is set equal to zero. If the member is straight and the post-tensioned wire is curved the equivalent load w_p should be used for determining moments, reactions, and shears. If the equivalent load w_p is based on an e which results from curvature of the post-tensioned wires as well as curvature of the axis of the beam, select any convenient datum above the wires to determine the e which should be used to compute the portion of w_p to be used in the calculations for reactions and shears.

A structure may be analyzed for applied loads and for the equivalent post-tensioning loads. The combined effect may be found by use of the principle of superposition.

Prestressed rigid frame

The prestressed rigid frame may be analyzed the same as any other reinforced concrete rigid frame, if first the equivalent load is determined.

Fig. 15 shows a two-hinged rigid frame with straight post-tensioned wires. It is assumed that no eccentricity exists at the ends and that e varies as a parabola. For the post-tensioning effect the frame would be analyzed for moments using all the loads shown. The forces indicated by dashed lines would be set equal to zero in determining the reactions and shears.

REMARKS

Thus far in the discussion friction has been neglected. This factor should

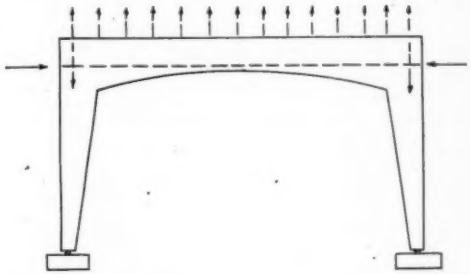


Fig. 15—Two-hinged rigid frame with straight post-tensioning wires

be small especially for straight wires. In any event every effort should be made to allow the wires to slip easily in the sheath.

The effect of shrinkage and plastic flow should be taken care of in the usual manner, *i.e.*, by reducing the working value of T. A 15 to 20 percent reduction is usually recommended.

The error involved in assuming a circle and a parabola to be equivalent might be a maximum of about 1 percent which may be neglected.

The effects of prestressed (or post-tensioned) wires may be expressed as equivalent loading conditions. It is shown that the equivalent load method can be most expeditiously handled. Also, the equivalent load method aids in visualizing the function of the post-tensioned wires.

In analyzing a prestressed concrete structure, first, compute the equivalent load. Second, apply this load to the structure, noting that a thrust exists at the end connections. Third, analyze the structure in any of the usual ways (using slope-deflection, moment distribution, etc., if the structure is statically indeterminate) to determine the bending moments. Fourth, in computing the shears, if the wires are straight set the equivalent loads equal to zero and use only the moments. If the wires are curved apply the equivalent load, resulting from such curvature, to the member.

The internal vertical reaction at the end is equal to the vertical component of the tension in the wires.

APPENDIX 1

NOTATION

The following letter symbols were used:

a = haunch
a_1 = area of M/EI diagram for $M_2 = 1$
A = area of M/EI diagram
d_c = depth of beam at center of span
d_x = depth of beam at distance x
e = eccentricity, usually at center of span
e_x = eccentricity at distance x
E = modulus of elasticity
I = moment of inertia
L = span length
M = bending moment caused by post-tensioning
M_2 = bending moment at support 2
M_x = moment at distance x caused by post-tensioning
P = axial compression in member due to prestress
P_p = equivalent concentrated load resulting from post-tensioning
r = radius of curvature
R_2 = reaction at support 2
Δ_x = elemental length of beam
T = post-tension in wires
w_p = equivalent distributed load resulting from post-tensioning
$\overline{x}$ = distance from support 1, toward support 2, to the centroid of the M/EI area in span 1-2 (for R_2 loading condition)
δ_2 = upward deflection at support 2 caused by post-tensioning
δ_2' = downward deflection at support 2 caused by downward reaction (or load) at 2
Σ = summation (over one span)

Title No. 48-29

Why Small Jobs Frequently Get Poor Concrete*

By A. W. BRUST†

SYNOPSIS

A survey of materials, methods of proportioning, mixing and placing ready-mixed concrete for housing and small industrial projects. Lack of systematic material and mix control and general absence of field inspection and testing result in great variations in concrete of supposedly uniform quality.

INTRODUCTION

"Good concrete is made of fine and coarse aggregate, cement and water; poor concrete is made of the same materials."

The writer doesn't know who first said this but he finds it an appropriate starting point from which to investigate some of the reasons for variations in properties and behavior of concrete of supposedly uniform quality.

The following remarks are based principally on personal observations in the St. Louis area during 1947-1950 together with a small amount of rechecking during 1951. This survey is limited almost entirely to concreting operations which, individually, are quite small but which, due to the large number of units involved, used a substantial quantity of ready-mixed concrete. Similar projects using job-mixed concrete were not included in the survey.

Any attempt to summarize in one package the operations of more than a score of mixing plants which in turn receive their materials from nearly a score of sources which are changed at frequent intervals would be both futile and misleading. It is the very diversity of methods and results that the writer attempts to lay before you for consideration and discussion.

Distinction should be made between plants located in the midtown area which principally serve municipal and major industrial projects with adequate engineering and inspection, and plants in outlying and suburban areas whose principal business is with individual contractors and builders operating without architectural or engineering service and inspection. It is this latter group with which these remarks are mostly concerned.

Such a distinction, however, cannot be sharply defined since most companies solicit jobs anywhere in the area and have their mixer trucks loaded at the most convenient batching plant that will sell to them. A considerable number of general materials companies without batching facilities are selling

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ready-mixed concrete by the simple expedient of acquiring one or more mixer trucks which are loaded for them by the concrete plant most convenient to the job. Thus Company A may sell, under its own name, concrete which has been batched by any of perhaps a dozen different plants. This practice added to an already highly competitive business points to the desirability of a more uniform code of practice of materials control, mix design and subsequent handling of ready-mixed concrete than now exists.

CEMENT

All major concrete plants use bulk cement except that high early strength cement, when used, is usually in sacked form. Any of four local brands are used almost interchangeably. Some plants show a preference for particular brands but most of them buy regularly from two or more sources as partial insurance against breaks in continuity of supply. Some difference of behavior between brands has been observed but this subject has not been investigated sufficiently to warrant further comment.

FINE AGGREGATE

Mississippi River sand is used more generally than other types although appreciable amounts of Meramec and Missouri River sands are also used. Sand from all sources is obtained by dredging. Due to selective deposition, Mississippi River sand dredged from one spot shows little variation in particle size. Shifting of dredging operations to different locations to obtain properly graded sand in the same barge load adds substantially to the cost and, quite naturally, is held to a minimum so long as the purchaser does not specify gradation but merely orders fine, medium or coarse sand.

Due to the considerable amount of lignite in Mississippi sand, Meramec and Missouri River sands are used extensively for floors and other finished surfaces because of the virtual absence of lignite in those materials. Meramec sand is quite angular but gradation is generally good. Concrete made with it produces harsher mixes for a given sand content. Missouri River sand from this area is typically too fine for most economical concrete.

The writer offers no arguments concerning the relative merits of the various sands available but merely points out that the variety bought in a highly competitive market under only the general designation of fine, coarse or medium sand without adequate inspection and control of fineness modulus and particle size distribution, together with proportioning methods to be discussed later, is of itself sufficient reason for much of the difficulty in obtaining concrete of uniform quality. Sieve analyses indicate that the fine sand ordered by one company and the medium sand ordered by another might well come from the same bin and either may be easily converted to coarse sand by adding a few percent of torpedo size.

With rare exceptions, sand sold on the local market under the general designations of "fine," "medium," or "coarse" sand is deficient in material retained on the number 8 sieve, deficient in minus 100 size and contains too

high a percentage of one size material in the 30-50 sieve area. The lack of sand coarser than the number 8 sieve usually causes no difficulty because the gravel used often contains an excess of minus 4 size to supply the deficiency but the virtual absence of minus 100 material is a major cause of poor workability in the lean concretes so generally used. Low fineness modulus which commonly ranges from 1.8 to 2.4 in unspecified sands contributes to the high water content required for workability.

COARSE AGGREGATE

Nearly all coarse aggregate used in concrete in the St. Louis area is so-called Meramec River gravel obtained by dredging from the river or from bank deposits on or near the river. Nine gravel plants are in constant operation at points along the river from a few miles west of Pacific, Missouri, to points several miles east of Fenton, Missouri. These plants ship principally by railroad. In addition, there are several trucking plants in part-time operation in the same locality.

The mineral character of the gravel from the various sources differs only slightly—being made up almost entirely of waterworn chert released by the disintegration of limestone and dolomitic formations. Particle size distribution, however, varies sharply from one locality to another as does the degree of contamination with small driftwood and other coarse organic material which is not removed in the screening and washing process. The amount of coated gravel and the degree of coating also shows large variations.

Those deposits farthest upstream and flood deposits farthest from the channel contain more large size material which needs to be crushed. Any considerable amount of crushed gravel adversely affects the workability and the sand/gravel ratio required in the concrete. Gravel from plants farthest down stream typically is deficient in the larger sizes and contains too much minus ½ in. material.

The foregoing account is not intended to be an adequate description of the gravel aggregate available in the area but shows why the concrete plants which frequently buy materials from several sources simultaneously find it impossible to maintain a uniform product when there is no check on either gradation or moisture content. When part of their aggregates come from well-drained stock piles and part is delivered by truck direct from the washing plant, these difficulties are aggravated considerably.

CONCRETE

Until very recently the great bulk of concrete in the area was sold as 1:3:5, 1:2:4, 1:2½:3½, etc. These designations, ostensibly indicating mix proportions, were actually only means of designating mixes selling at different prices. There is no assurance that 1:3:5 concrete sold by Company A bears more than a general resemblance to 1:3:5 concrete from Company B. Since the cost and hence the selling price is determined almost entirely by the cement content, there was a sort of general agreement that the mix designation also indicated the cement content.

In 1947 when this survey was begun, 1:3:5 concrete nearly always meant concrete containing 4¼ sacks cement per yd, 1:2:4 concrete variously contained 5¼, 5⅓ or 5½ sacks per yd. Water content and sand-gravel ratio were and still are varied to suit the individual customer. By 1949 the use of 1:3:5 concrete had become much more general in out-lying small jobs and the cement had decreased to 4 sacks. In some instances where dispersing or air-entraining agents were used, cement factor was reduced to 3.8 sacks which seemed to be the irreducible minimum with which to make salable concrete, using ¾-in. maximum size aggregates.

The trend in 1951 is to discard the designation of concrete by mix proportions and to price it to the customer on the basis of cement content. Thus concrete containing 4 sacks per yd and mixed approximately in the proportions of 1:3.7:4.9 by weight and 41 to 43 gal. total water per yd last year usually was called 1:3:5. Today it is more likely to be called 4-sack concrete.

The substitution of cement factor, instead of misleading and incorrect mix proportions as a means of designating quality of mix, is a step in the right direction. Further controls are needed before ready-mixed concrete takes its rightful place as an economical manufactured building material of predictable, uniform quality.

Although conditions vary somewhat from plant to plant and from job to job, the following conditions were quite general at the 17 plants investigated by the writer in the summer of 1950:

1. The St. Louis ready-mixed concrete companies, following what is considered ordinary competitive business practice, believe that their function is to furnish concrete batched to the order of the customer or his representative. Little regard is given to minimum strength properties or the suitability of the quality ordered for the intended use. The customer presumably knows what he wants and will buy it elsewhere if they decline to furnish it. Concrete sold on a guaranteed strength basis is the exception, rather than the rule.
2. Few companies systematically test their materials or require suppliers to do so.
3. Delivery tickets normally show only the number of yards and either the cement content or an arbitrary designation such as 1:2:4. Amount of water and actual amounts and proportions of aggregates are not shown.
4. Batch weights are commonly computed on an assumed fixed amount of moisture in the aggregates. Sand is usually assumed to contain 3 percent free moisture and gravel from 0 to 1 percent. Changes in batch weights of solid materials to allow for changes of moisture content in aggregates are occasionally made but more generally are not. Changes in slump desired by the customer are accomplished by regulating the amount of added water.
5. Both cement and aggregates are often purchased from any of several sources and no allowance is made for characteristic differences.
6. Sequence of batching varies from dumping completely mixed concrete from a plant mixer into an agitator truck to adding the various ingredients from separate bins to a mixer truck before mixing is begun. Thoroughness of mixing in a specified time varies as widely as methods of batching and types of mixers used.
7. Poorly mixed concrete is found only occasionally. When found it is almost always associated with very short hauls and overloaded trucks.
8. The use of air-entraining agents varies greatly. Some plants always add air-entraining agents unless specifically instructed not to, while others use them only under protest. The use of air-entraining concrete is increasing rapidly at the present time.

9. Methods of computing yield vary considerably. One large company bases its calculations on the assumption that a cubic yard of concrete weighs 4000 lb; a second company uses the assumption that a yard of concrete should contain 1400 lb of fine and 1900 lb of coarse aggregate; a third arbitrarily adds about 200 lb of sand per yard over and above his computed mix to avoid any chance of short-changing the customer. Many of the smaller, newer companies depend on the cement salesman to compute their mixes for them. So far the writer has found no two of them to agree exactly on methods of computing yield or the quantities necessary to produce a cubic yard of concrete. Significance of the "absolute volume" of ingredients in determining batch quantities is not widely understood.

CONCRETE PLACING PRACTICES

Bad

Sectional plywood wall forms are used more generally than other types. These are either owned or rented by the builder, depending on the scope of his operations. In either case, form costs demand that they be used as continuously as possible. It was found common practice to erect forms in the morning, pour concrete in the afternoon, strip forms and re-erect them on previously poured footings the following morning, thus enabling a builder to pour a foundation wall every 24 hours with one set of forms. With ten or more gallons of water per sack, four-sack concrete when 16 to 20 hours old is pretty fragile stuff. Complete your own mental picture of the effect of stripping forms under these conditions.

In general concrete literature, the process of filling the forms is usually referred to as "placing." The more generally used term "pouring concrete" is far more descriptive. When the terrain will permit, truck access to the forms is provided at several points. Frequently this is possible only at considerable extra expense. The entire foundation wall then is often poured by chute from one corner of the site and allowed to run from 30 to 50 ft along the forms by gravity together with such additional manual aid as may be necessary. Cases have been noted where concrete has flowed forty feet without assistance with only nominal segregation. In another case, just after the discharge of a truckload of concrete, 20 in. of water were found on top of the concrete which had found its way to the other side of the foundation. The water was only 20 in. deep because holes in the form at that height prevented its accumulation to a greater depth.

On one project of 150 house foundations the only contractor's personnel on hand when concrete was delivered was one laborer whose sole activity at the time of my visit was to receive the delivery ticket and place a splash board to guide the concrete into the forms. Presumably some striking off and leveling became necessary when the forms were nearly full but at least the first three loads found their way to their final resting point without assistance. Test cylinders taken from the truck on this job showed a 28-day strength of 1007 psi under laboratory curing and 1295 psi for jobsite curing. Perhaps the high rate of evaporation on the job got rid of a little of the excess water before the concrete set.

Good

Along with these cases of flagrant malpractice there are other instances where the builder does a competent and conscientious job of producing good concrete work. At the same time and only a few blocks from the instance cited above, a second builder was putting up a group of houses of identical foundation plan and under the same basic specifications. He was using 5-sack concrete instead of four and rigidly holding the slump between 4 and 6 in. His forms were filled from six chute locations instead of one. A competent foreman was constantly on the job during delivery and two or more laborers were doing a good job of spading and compacting the concrete.

JOB CHECKS

After becoming somewhat familiar with plant practices at 17 batching plants and most of the aggregate plants from which they received their supplies, a limited field testing program was carried out in July and August, 1950.

Housing jobs at seven locations were selected. All were operating under the same basic specifications, using so-called 1:3:5 concrete for footings, walls, floors, steps and driveways. Five contractors, 4 ready-mix companies and 4 aggregate sources were represented. Results of strength tests at 7 and 28 days are shown in Table 1.

The specifications provided for a minimum mix consisting of 1 part cement, 3 parts sand and 5 parts coarse aggregate. Whether the parts specified were by weight or volume was not stated. The specifications stated further that the water content shall not exceed 7½ gal. of water per bag of cement and that transit mix concrete shall have minimum compressive strength of 2000 psi at 28 days. No aggregate gradation was specified nor was there any limitation placed on consistency or slump.

On the seven jobs checked, the strength obtained at 28 days in no case was as great as 2000 psi under either laboratory or job site curing, but ranged from 45 percent to 80 percent of that figure. The over-all average of 14 cylinders, half laboratory cured and half job cured was 1358 psi or 68 percent of 2000 psi. The average seven day strength was 712 psi.

TABLE 1—STRENGTH OF SMALL-JOB CONCRETE

Job	Use	Source	Contractor	7-day strength, Lab. cure, psi	28-day strength, psi	
					Lab. cure	Job cure
1	Driveway	1	1	936	1544	1609
2	Walls	2	2	854	1566	1348
3	Footings	3	3	818	1586	1484
4	Walls	2	2	705	1400	1427
5	Footings	1	4	689	1372	1363
6	Walls	4	5	512	1007	1295
7	Walls	1	2	467	1119	896
			Ave.	712	1370	1346

Under the specification of not more than 7½ gal. per sack, only 30 gallons per yard is permitted in 4-sack concrete or 37½ gal. when using 5 sacks of cement per cu yd. Anyone who has ever made a trial mix of concrete containing 4 or 5 sacks per yd and using natural sand and ¾-in. gravel from this area, will recognize the futility of attempting to obtain workable concrete with a slump of 4 in. or more without using at least 38 gal. water per yd with plain cement or about 36 gal. with air entrainment. Unless the aggregates are very carefully graded and the sand content is held to a minimum for workability, the slump obtained with these amounts of water is usually less than 2 in. This clearly is a case of a poorly written specification practically impossible to meet.

Slumps were taken but not reported in Table 1. All were over 6 in. except Job 1 which was a driveway on a fairly steep slope.

Unfortunately it was not feasible to determine the actual amount of water used in concrete on these seven jobs. Moisture tests made on other jobs clearly indicated that concrete of the mix and consistency reported in Table 1 requires more than 40 gal. per yd—a water-cement ratio in excess of 10 gal. per sack. The strengths obtained when plotted on a conventional water-cement versus strength chart indicate W/C between 10 and 13 gal. per sack.

The writer has found no cases of deliberate corner cutting but numerous cases where the builder or his representative disclaimed all knowledge that specifications limited the water content or specified minimum strengths and that he did not indicate either maximum water content or required strength in his order for concrete. There is every reason to believe that the contractor in the great majority of cases got exactly what he ordered from the ready-mix company—namely, the cheapest concrete available, mixed to the foreman's idea of proper fluidity. In cases such as this the contractor has been known to infer that the concrete plant forgot to put in some cement; the concrete company has retorted that water was added on the job; both contractor and concrete company ask pointed questions of the cement salesman concerning the quality of his product.

Three St. Louis concrete companies estimated that 75 to 90 percent of their 1950 sales were 4-sack concrete, sold under the designation 1:3:5. One company stated that they sold no concrete containing less than 4¼ sacks per yd; other companies declined to comment. There is little doubt in the writer's mind that the results shown in Table 1 constitute a fair cross section of most of the 4-sack concrete used in housing in the St. Louis area during 1947-1950.

IMPROVEMENT FORESEEN

The outlook for future improvement is definitely encouraging. A local builders association has argued in the past that 2000 psi concrete is not necessary to support the light loads imposed by one- or even two-story buildings. However, there is a growing realization that the function of a basement wall is something more than to support the vertical loads imposed and that the

addition of an extra sack of cement together with sufficient water control to prevent severe segregation pays off in better appearance, easier placement, and far fewer cracked and leaky basements, together with a bonus of nearly 100 percent in strength. The extra cost of 5-sack concrete instead of 4 is about $30 in the average 4 or 5 room house.

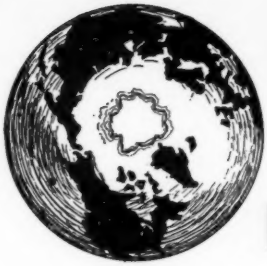
A partial recheck of the ready-mix companies investigated last year shows that considerably more 5-sack concrete is being used now instead of the 4-sack concrete which constituted so large a part of the previous year's sales. Furthermore, some companies are making a real effort to acquaint their customers with the general features of the water-cement ratio law and to set a limit on the water to be added to a mix. More companies have instructed their drivers to report to the plant when additional water is added at the job.

Considering the mass of publications during the past 30 years together with educational programs and demonstrations in the schools and on the job, it is hard to understand why anyone responsibly concerned with the specification, sale or placing of concrete should not be familiar, at least in a general way, with the interrelations between aggregate gradation, water content and consistency, or between water-cement ratio and such things as strength, watertightness, shrinkage and durability. The writer has long been inclined to attribute the use of over-lean, over-wet mixes to the urge to make a fast dollar but his more recent observations and conversations on the job lead him to believe that much of the trouble is due to lack of supervision and just plain ignorance on the part of those actually doing the job.

There still is need for extensive education at the elementary level, education all along the line from the specification writer and supervising architect to the foreman and concrete finisher. The American Concrete Institute aided materially in improving the quality of concrete in major structures where adequate specifications, inspection and testing are an integral part of the contract. Perhaps the Institute can do an even more outstanding job of education and improvement in the small job field which, in the past, too often has been functioning with inadequate specifications or inspection and under the direction of the labor foreman and the concrete finisher instead of the concrete technologist.

A very short essay written by John Ruskin long before the advent of ready-mixed concrete seems to be equally appropriate today. He wrote:

> It's unwise to pay too much, but it's unwise to pay too little. When you pay too much you lose a little money, that is all. When you pay too little, you sometimes lose everything, because the thing you bought was incapable of doing the thing you bought it to do. The common law of business balance prohibits paying a little and getting a lot. It can't be done. If you deal with the lowest bidder, it's well to add something for the risk you run. And if you do that, you will have enough to pay for something better.



Letters from Readers*



BY WAY OF SYNOPSIS

C. D. NORTON adds some information concerning cracks in tunnel lining.

L. RUDCHENKO and W. E. MCNELY offer a method of determining fixed end moments in beams without the use of handbooks.

GORDON W. GREEN gives a method for quickly determining the cement factor of a batch of concrete.

Cracks in Tunnel Lining (LR 48-10)†

Consultation at long distance is of doubtful value, but the following thoughts may prove of interest. Is there by chance any sulfide present in the aggregate, or can such sulfide be precipitated into the concrete when it is forced into the forms? How much does the roof settle during 24 hours? Set a dial indicator on a post under the forms and measure the deflection. Try a rapid hardening cement such as "Ciment Fondu," which normally attains a strength of 6000 psi in 24 hours, using 6 bags to a yard.

C. D. NORTON, engineer,
Montreal, P. Q., Canada

Determination of Fixed End Moments in Beams with Irregular Loading Without Use of Formulas from Handbooks (LR 48-12)‡

The method of determining fixed-end moments in beams with irregular loading without use of formulas from handbooks, as presented by Joseph Zalkin,‡ is an interesting and obviously correct solution to the problem. However, there are other methods of approach which are either simpler and as accurate, or more accurate and as simple as the method demonstrated.

One objection that might be raised to the solution presented is that the formulas for fixed-end moments actually used are not necessarily known without handbooks. While most engineers would know the values of fixed-end moments for spans subjected to uniform and concentrated loads, it is doubtful that many would know the formula for a span under triangular loading without referring to a handbook.

For the purposes of ordinary design calculations, undue accuracy is neither warranted nor desired, due to the inaccuracies

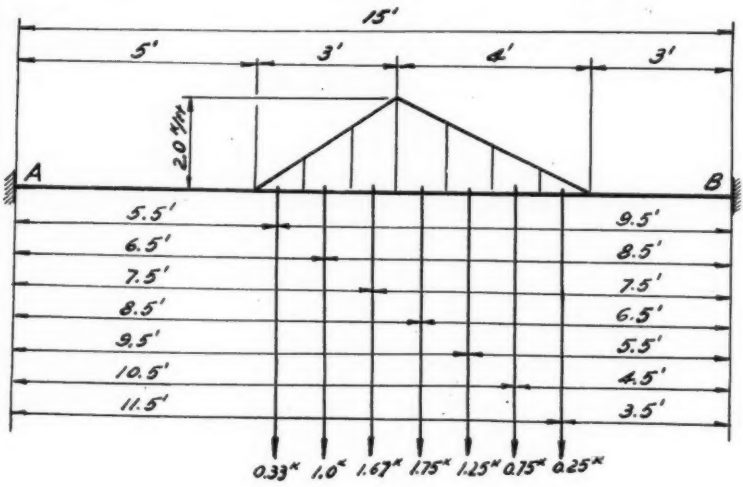
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†See also ACI JOURNAL, Oct. 1951, *Proc.* V. 48, p. 185.

‡See also ACI JOURNAL, Oct. 1951, *Proc.* V. 48, p. 186.

with which loads are predicted and the limited time available for such computations. Therefore the most desirable solution is one giving a reasonable amount of accuracy with minimum effort and possibility of error.

Such a solution to the example given is offered, using only one formula—the one for a concentrated load at any point on a span. The triangular loads are divided into one-foot segments and the incremental loads considered concentrated at the center of each segment. Then, by superimposing the moments due to the individual concentrated loads, the fixed-end moments are obtained. The complete solution is as follows:



Pab^2	Pa^2b
$0.33 \times 5.5 \times 9.5^2 = 165$	$0.33 \times 5.5^2 \times 9.5 = 96$
$1.0 \times 6.5 \times 8.5^2 = 470$	$1.0 \times 6.5^2 \times 8.5 = 359$
$1.67 \times 7.5 \times 7.5^2 = 704$	$1.67 \times 7.5^2 \times 7.5 = 704$
$1.75 \times 8.5 \times 6.5^2 = 630$	$1.75 \times 8.5^2 \times 6.5 = 824$
$1.25 \times 9.5 \times 5.5^2 = 360$	$1.25 \times 9.5^2 \times 5.5 = 622$
$0.75 \times 10.5 \times 4.5^2 = 159$	$0.75 \times 10.5^2 \times 4.5 = 372$
$0.25 \times 11.5 \times 3.5^2 = 35$	$0.25 \times 11.5^2 \times 3.5 = 116$
$\Sigma Pab^2 = 2523$	$\Sigma Pa^2b = 3093$

$$FEM_A = \Sigma Pab^2/L^2 = 2523/225 = 11.21 \text{ kip-ft}$$
$$FEM_B = \Sigma Pa^2b/L^2 = 3093/225 = 13.75 \text{ kip-ft}$$

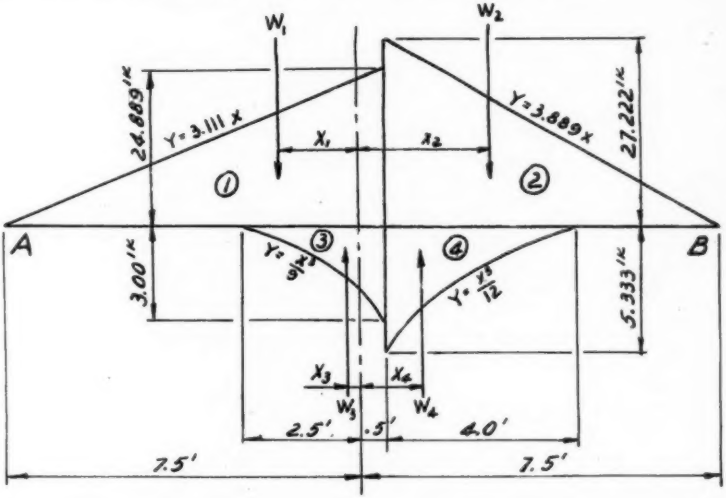
Should greater accuracy be desired the method of column analogy can be applied as indicated by Mr. Zalkin. However, this method can be used more simply with integration than with superposition, thereby resulting in an exact analysis rather than an

approximation. As a check on the previous solution, the beam is treated as being simply supported and the moment diagram drawn by parts. Then the areas and centers of gravity of the areas under the moment curves are found, a process requiring no more difficult integration than $\int x^n dx$.

Reactions for simple beam:

$R_A = (4 \times 5.67 + 3 \times 8)/15 = 3.111^k$

$R_B = 7.0 - 3.111 = 3.889^k$



Area No.	x	W	M(=Wx)
1	$\frac{-8.0}{3}$ + 0.5 = -2.167	-24.889 x 4.0 = -99.556	+215.703
2	$\frac{7.0}{3}$ + 0.5 = +2.833	-27.222 x 3.5 = -95.278	-269.951
3	$\frac{-3.0}{5}$ + 0.5 = -0.100	0.25 x 3 x 3 = +2.250	-0.225
4	$\frac{4.0}{5}$ + 0.5 = +1.390	0.25 x 4x5.333= +5.333	+6.933
		ΣW = -187.251	ΣM=-47.540

Analogous column:

$A = 15$; $I = 15^3/12 = 281.25$; $c = \pm 7.5$

$W = -187.251$; $M_i = -47.540$

$$FEM_A = \frac{-187.251}{15} + \frac{(-47.540) \times (-7.5)}{281.25}$$

$$= -11.215 \text{ kip-ft}$$

$$FEM_B = \frac{-187.251}{15} + \frac{(-47.540) \times 7.5}{281.25}$$

$$= -13.751 \text{ kip-ft}$$

The results shown indicate that very nearly identical results are obtained by using the approximate method, with slide-rule accuracy, and the exact analysis by column analogy. This would suggest that even larger segments could have been used, simplifying the superposition process and still retaining the necessary accuracy required by design calculations.

L. Rudchenko and W. E. McNely, Kaiser Engineers, Oakland, California

Cement Factor Determination (LR 48-18)

The actual cement factor of a concrete mix will vary from the designed value when variations occur in the total water requirements of the mix or if the amount of entrained air does not equal exactly the amount provided for in the design.

A quick and convenient method of determining the actual cement factor by use of a single equation is outlined here. One physical measurement, the weight of the fresh concrete per cu ft, is required. This weight may be measured when test cylinders are made or when a slump test is taken. The accuracy of this method depends upon the accuracy of the unit weight determination and will be affected by improper sampling, mixing, or inaccurate batching. It is imperative also that weights of the ingredients used in the mix be correctly ascertained.

The equation is derived from four basic equations used frequently in concrete mix design.

$$V = Wt/U \quad (1)$$
$$Y = V/N \quad (2)$$
$$N = W_c/94 \quad (3)$$
$$CF = 27/Y \quad (4)$$

Where

V = Volume of concrete per batch in cu ft
Wt = Total weight of all ingredients in lb
U = Unit weight of concrete, lb per cu ft
Y = Volume of concrete produced per sack of cement, cu ft
N = Number of sacks of cement in batch
W_c = Weight of cement in batch, lb
CF = Cement factor in sacks of cement per cu yd of concrete

Substituting Eq. 2 in Eq. 4

$$CF = 27N/V \quad (5)$$

Substituting Eq. 1 in Eq. 5

$$CF = 27NU/Wt \quad (6)$$

Substituting Eq. 3 in Eq. 6

$$CF = 27UW_c/94Wt \quad (7)$$

Reducing 27/94 to decimal fraction

$$CF = 0.2872\ UW_c/Wt \quad (8)$$

In Eq. 7 when Wt is the batch weight of a design cu yd, the equation is equivalent to dividing the true weight of 27 cu ft of concrete by the designed batch times the sacks of cement in the batch.

Calculations may be made quickly and easily in field or laboratory using a slide rule. A 10-in. rule is sufficiently accurate for all work.

Examples

1. The following mix is designed to yield 1 cu yd of concrete containing 6 sacks of cement per cu yd at a 3-in. slump.

Gravel	2160 lb
Sand	1140 lb
Cement	564 lb
Water	236 lb
	4100

What is the cement factor when 33 lb additional water are added to the concrete to make a 7-in. slump? The measured unit weight is 150.1 lb per cu ft. Substituting in Eq. 8

$$CF = \frac{0.2872 \times 564 \times 150.1}{4100 + 33} = 5.88 \text{ sacks}$$

per cu yd.

2. The following mix is designed to yield 1 cu yd of concrete containing 5 sacks of cement per cu yd at a 4-in. slump and with 4.5 percent entrained air.

Gravel	1940 lb
Sand	1260 lb
Cement	470 lb
Water	230 lb
	3900

The actual amount of air is determined to be 3.5 percent and the unit weight is 145.8 pounds per cu ft. What is the cement factor? From Eq. 8

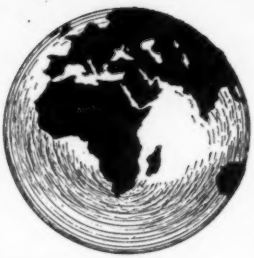
$$CF = \frac{0.2872 \times 470 \times 145.8}{3900} = 5.05 \text{ sacks}$$

per cu yd.

GORDON W. GREENE, district testing engineer, Pacific Coast Aggregates, Inc., Fresno, Calif.



Current Reviews*



of Significant Contributions in Foreign and Domestic Publications

Industrialized construction (La costruzione industrializzata)

Il Cemento (Milan), V. 48, No. 11, Nov. 1950, pp. 200-203

Reviewed by G. MIANULLI

Describes how the acute housing shortage in Italy is being gradually overcome. The solution is a reinforced concrete prefabricated house, designed and built by S.A.C.C. A.I., a firm specializing in reinforced concrete structures.

Studies in cement-aggregate reaction XVI—The effect of hydroxyl ions on the reaction of opal

H. E. VIVIAN, *Australian Journal of Applied Science* (Melbourne), V. 2, No. 1, Mar. 1951, pp. 108-113

The severe attack on opal by sodium hydroxide and by tetramethyl ammonium hydroxide is discussed and contrasted with the lack of attack by trimethylamine, alkali salts and calcium hydroxide. The evidence is strong that attack of opal is caused by hydroxyl ions.

A large jetty on Southampton water

Concrete and Constructional Engineering (London), V. 46, No. 9, Sept. 1951, pp. 269-274

Reviewed by GLENN MURPHY

A 2000-ft approach and a 3200-ft pierhead constitute the new marine terminal of the Esso Petroleum Co., Ltd., at Fawley. About 30,000 cu yd of reinforced concrete were used in the construction, which includes 637 rectangular piles and 503 cylindrical piles, the latter having an outside diameter of 32 in. and an inside diameter of 20½ in. Caissons of 36 ft and 40 ft outside diameter are used. General design features are described.

Iterative procedure for the solution of a continuous beam fixed at its supports (Procedimento iterativo per la risoluzione della trave continua solidale con gli appoggi)

V. COMIZ, *Il Cemento* (Milan), V. 47, No. 11, Nov. 1950, pp. 195-196

Reviewed by GENNARO MIANULLI

This approximate method was described in *Il Cemento*, No. 6, 1950. In the present installment, the procedure is applied to continuous beams with fixed ends. A practical example is included.

Precast concrete north-light roof trusses

Concrete and Constructional Engineering (London), V. 46, No. 9, Sept. 1951, pp. 278-282

Reviewed by GLENN MURPHY

Precast units used for framing the roof consist of L-shaped members with legs approximately 17½ ft and 28½ ft long to span 33 ft 4 in. The short (steep) leg is 13½ in. deep while the flat leg is 15 in. deep. Both are 6 in. wide. The frames rest on precast concrete columns, which were made on the site and erected with the aid of an 8-ton mobile crane. Each precast frame weighed 2½ tons.

Raising roofs by hydraulic jacks lowers construction costs

STANLEY P. STEWART, *Technical Bulletin* No. 17, Housing and Home Finance Agency, June 1951, pp. 1-6

General description of the Youtz-Slick method of concrete slab construction in which roof and floor slabs are cast at ground level and raised on steel columns to final position. Practical application at Trinity University is discussed with some references to the economy of the system.

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Three new systems of prefabrication at Le Harve (Trois procedes nouveaux de prefabrication au Havre)

J. P. ROJOT, *Cahiers du Centre Scientifique et Technique du Batiment*, V. 11, No. 106, First Quarter, 1951
Reviewed by PHILLIP L. MELVILLE

In view of the large amount of war destruction, lack of skilled labor and shortage of local materials, large scale experiments were undertaken using new building materials. The article describes three methods —concrete blocks of special section, precast units of different sizes, and precast slabs.

New cement works at Shoreham

Concrete and Constructional Engineering (London), V. 46, No. 9, Sept. 1951, pp. 287-292
Reviewed by GLENN MURPHY

The new cement plant of the British Portland Cement Manufacturers in Sussex includes two 350-ft kilns and auxiliary works, constructed in a chalk quarry. The kilns are 10 ft in diameter and are supported on reinforced concrete piers. The eight bunkers, the twelve 60 x 26½-ft diameter silos, the 300-ft chimney and the slurry plant are described. Electrostatic dust precipitators are used in the kiln house.

Multiple-story buildings with precast walls at London Airport

Concrete and Constructional Engineering (London), V. 46, No. 9, Sept. 1951, pp. 275-277
Reviewed by GLENN MURPHY

Six new buildings are described, four of them being 2-story office buildings. The exterior wall sections are 7½ in. thick, 5 ft wide and 8 ft 3 in. or 11 ft 3 in. high, and were transported from the factory to the site on trucks. The columns and floor and roof slabs were cast in place. Beams were precast in 8 ft 10 in. lengths.

Bending moments due to temperature changes in continuous reinforced concrete structures

ALBIN CHRONOWICZ, *Concrete and Constructional Engineering* (London), V. 46, No. 8, Aug. 1951, pp. 241-251
Reviewed by GLENN MURPHY

The author develops a procedure for evaluating the fixed-end moments in reinforced concrete beams subjected to temperature differentials. The conventional assumptions are made, and the application to the evaluation of the stresses in rigid frames through the use of moment distribution is indicated. Several illustrative problems are included.

Manufacture of slag-cement by the wet process (Preparation du ciment de laitier par voice humide)

M. MARY, *Annales de l'Institut Technique du Batiment et des Travaux Publics*, New Series, No. 200, July-Aug. 1951
Reviewed by PHILLIP L. MELVILLE

The addition of slag finely ground by the wet process gave very good results in the construction of a dam in southwestern France. Acceptance was based on preliminary tests. Very uniform results were obtained with slag coming from one given blast furnace. The ground slag had good cementing properties. It is expected that further use of slag would result in appreciable savings.

Portland cement paint

Bulletin ACI, New Zealand Portland Cement Assn., Wellington, 1951, 16 pp.

Recommendations on the composition and application of cement base paints. Not materially different from "Recommended Practice for the Application of Portland Cement Paint to Concrete Surfaces (ACI 616-49)."

Lightened concretes—III and IV (Les betons alleges)

J. P. LEVY, *Revue des Materiaux*, No. 430, July 1951, pp. 236-239
Reviewed by PHILLIP L. MELVILLE

In the third part of a comprehensive study, the author reviews the use of wood fibers in lightweight concrete. The importance of using the proper fibers and treatment before use is stressed. In the fourth part, an overall review of advantages and disadvantages of the sundry methods is presented. It is concluded that lightweight mineral aggregate concrete has tremendous possibilities.

A prestressed concrete lighthouse

Concrete and Constructional Engineering (London), V. 46, No. 8, Aug. 1951, pp. 257-258
Reviewed by GLENN MURPHY

The reconstructed Berck lighthouse near Boulogne consists of a prestressed concrete cylinder with an external diameter of 18 ft 4½ in. and a height of 106 ft, resting on a monolithic concrete plinth 18½ ft high above the ground level. The shaft consists of 36 precast segments per course with four sets of nine vertical wires, and a precast concrete ring every third course. The Freyssinet system of prestressing was used.

Residential flats in South London

Concrete and Constructional Engineering (London), V. 46, No. 8, Aug. 1951, pp. 253-256
Reviewed by GLENN MURPHY

Some features of the construction of six blocks of reinforced concrete flats at Clapham are described. One is the lack of expansion joints in the 130 to 170-ft lengths of building. Except for the outer walls, which are brick, the structural members are of reinforced concrete. Part of the stairs and the floor units are of precast reinforced concrete.

Load carrying problems in reinforced concrete construction (Traglastprobleme des Stahlbetonbaues)

A. HABEL, *Beton-u. Stahlbetonbau* (Berlin), V. 46, No. 1, Jan. 1951, pp. 6-10, No. 2, Feb. 1951, pp. 32-35
Reviewed by RUDOLPH FISCHL

This paper, which gives partial results of a longer investigation, deals with buckling of elastically restrained columns, influence of the arrangement of the reinforcing, influence of prestressing and shrinkage on the buckling of columns, ultimate movements in rectangular sections with compressive reinforcement and the safety against failure of continuous beams with variable loading. Twenty figures illustrate the paper.

Winter construction—ABC (Vinterbyggeriets—ABC)

The Danish National Institute of Building Research (Copenhagen), 24 pp.

An attractively prepared and illustrated booklet containing suggestions for the contractor for building construction in winter. Protection of materials and workmen, site precautions, temporary heating of buildings under construction, concreting methods and curing at low temperatures are discussed from the practical standpoint. The economic justification of the added cost of winter construction is emphasized.

The slab supported on two sides (Die zweiseitig gelagerte Platte)

HUGO OLSEN and FRITZ REINITZHUBER, Wilhelm Ernst & Sohn, Berlin, V. 2, 1951, 178 pp. 32 DM.
Reviewed by ARON L. MIRSKY

This volume, subtitled Applications and Conclusions, is an extension of the first volume (1950). After a discussion of slabs supported on two sides, there are presented six fully-worked numerical examples of reinforced-concrete slabs (mainly bridge slabs) under various conditions of loading, followed by 60 pages of curves and tables of bending moments, etc., for various ratios of slab length to width, for the convenience of designers.

Studies in cement-aggregate reaction XVII—Some effects of temperature on mortar expansion

H. E. VIVIAN, *Australian Journal of Applied Science* (Melbourne), V. 2, No. 1, Mar. 1951, pp. 114-122
AUTHOR'S SUMMARY

Elevated storage temperatures increase the rates of aggregate reaction and of water absorption. At the same time, however, the quantity of reaction product that is sufficiently rigid to exert a directional swelling pressure on the mortar and thus widen cracks is rapidly decreased. Thus, although elevated temperatures increase the initial rate of mortar expansion, they decrease total expansion.

Application of Cross' method for beams with variable section modulus (Anwendung des Cross-Verfahrens auf Staebe mit veraenderlichem Traegheitsmoment)

R. OBRACZAY, *Oesterreichische Bauzeitschrift* (Vienna), V. 6, No. 1, Jan. 1951, pp. 1-3
Reviewed by RUDOLPH FISCHL

Author seems to be somewhat confused about the basic principles of moment distribution. What he calls a simplified Cross method is strictly a slope-deflection iteration method, developed by others, and has nothing to do with Cross' method. Using the tables from Guldan's book *Frames and Continuous Beams* to determine the factors for the angular deformation, the iteration is done in the usual way. One example illustrates the method.

Non-destructive testing of concrete

R. JONES, *The Reinforced Concrete Review* (London), V. II, No. 5, Jan. 1951, pp. 315-328
Reviewed by C. P. SIESS

Method described is measurement of longitudinal wave velocity by electronic timing of supersonic radar-type pulse. Apparatus used has range up to 8 ft. Laboratory test data given to show correlation between compressive strength of cubes and wave velocity. Other laboratory applications include studies of setting phenomena, freezing and thawing, and determination of Poisson's ratio from longitudinal wave velocity and natural frequency measurements. Field applications also described briefly.

Wharf and tunnels at electricity works

Concrete and Constructional Engineering (London), V. 46, No. 8, Aug. 1951, pp. 233-240

Reviewed by GLENN MURPHY

A 290-ft wharf in the Thames is designed to accommodate 500-ton tankers delivering oil to the new Bankside power station of the British Electricity Authority. Most of the structure is supported on piers consisting of 6 ft 8 in. diameter reinforced concrete pipes filled with concrete and resting on groups of four 12-in. square precast reinforced concrete piles 45 ft long. The deck, which is 38 ft wide, consists in general of a 12-in. reinforced concrete slab.

Tunnels, through which cooling water is taken from and discharged into the river, are a part of the structure. They are 10 ft in diameter and are constructed of cast iron segments lined with concrete.

Construction details are given, and photographs and drawings of the structure are included.

Fire tests of steel columns protected with siliceous aggregate concrete

NOLAN D. MITCHELL, *Building Materials and Structures Report* 124, National Bureau of Standards, May 25, 1951. 12 pp. $.15

AUTHOR'S SUMMARY

Results of fire-endurance tests of four 6-in. steel H-columns protected with concrete made with highly siliceous aggregates are presented. The concrete was 2-in. thick around the column shaft and filled the re-entrant spaces. The aggregates investigated were river gravel and crushed quartz.

The columns were subjected to load throughout the tests. The loads on three columns, of magnitude as computed by a standard formula for allowable load, were constant. One column sustained a double load throughout the earlier part of the test.

The fire-endurance limits ranged from 2 hours 50 minutes for a column with crushed quartz aggregate concrete to 3 hours 34 minutes for one with gravel aggregate concrete. The results were consistent for columns protected with gravel aggregate concrete, and within reasonable limits for columns covered with quartz aggregate concrete.

The results of the column tests with respect to lack of spalling of the concrete were confirmed by tests on concrete wall slabs made with the same or like aggregates.

Laws controlling relations between specific surface and the technological properties of cement (Les lois regissant les liaisons entre la surface specifique et les proprietes technologiques de ciment

MICHEL JASPERS, *Revue des Materiaux*, No. 429, June 1951, pp.169-174, No. 430, July 1951, pp. 207-215, No. 431-432, Aug.-Sept. 1951, pp. 254-257

Reviewed by PHILLIP L. MELVILLE

The control of cement is based on the study of chemical, physical and technological variables. The variables may be independent such as mineralogy and specific surface or functions such as time of set and hardening, volumetric variations, heat of hydration, workability, segregation and durability. In this study, the mineralogical composition was assumed constant and the specific surface was taken as a variable. Rosin, Rammler and Sperling's equation giving the gradation of a ground product was used. Mathematical and graphical relations were established and it is stated that the properties of the cement and the strength of the resulting concrete may be immediately found by determination of the fineness of two samples of the cement ground to different specific surfaces. A bibliography of nine references is addended.

Techniques used in the experimental stress analysis of reinforced concrete structures

J. N. THOMPSON, *Proc., Society for Experimental Stress Analysis*, V. 8, No. 2, 1951, pp. 111-116

APPLIED MECHANICS REVIEWS Sept. 1951 (Holley)

Author describes applications of SR-4 resistant wire strain gages. Of particular note is the ingenious use of these gages, mounted internally, to detect the strains of a reinforcing bar. Author accomplishes this application by planing two circular bars down to mating semicircular sections, milling a rectangular groove in each, mounting a number of gages in each, and tack welding the sections together to re-form the desired rod. Technique for fastening and protecting gages and leads is described in adequate detail. Author states that gages with $\frac{1}{8}$-in. or $\frac{1}{4}$-in. gage lengths were unstable under repeated or dynamic loadings and over extended periods of time. Gages with 13/16-in. gage lengths were satisfactory under these conditions.

There is included a brief description of techniques found to be successful in the application of SR-4 gages to the surface of concrete and tile.

Substructure and oil store at electricity works

Concrete and Constructional Engineering (London), V. 46, No. 9, Sept. 1951, pp. 263-268
Reviewed by GLENN MURPHY

The new Bankside power house under construction on the Thames involves an appreciable amount of concrete. This article contains a description of the gravity retaining wall surrounding most of the structure, the foundation and floors, and access road. The associated wharf was described in a previous article. Concrete reinforced with expanded metal forms the base for the three million-gallon oil storage tanks. Reinforced concrete boxes with 3-ft thick walls are used as buttresses for the tank structure, which is covered with a flat slab reinforced concrete roof.

Valved-tremie applied to subaqueous concrete structures

A. BYRON HUNICKE, *Journal of the Franklin Institute*, V. 252, No. 2, Aug. 1951, pp. 105-136

Traces recent developments in types of tremie pipe used to deposit concrete in water and the method of operating them. Summarizes research with valves to eliminate the hazards of the old open tremie pipe and describes one which makes an elongated pneumatic caisson of the tremie. This was used extensively and with success in placing over 300,000 cu yd of concrete.

The routine of maneuvering such a tremie is described and the sequence, size and location of monoliths poured by this method are shown. Test results from cylinders drilled out of the finished work are discussed.

Studies in cement-aggregate reaction XVIII—The effect of soda content and of cooling rate of portland cement clinker on its reaction with opal in mortar

C. E. S. DAVIS, *Australian Journal of Applied Science* (Melbourne), V. 2, No. 1, Mar. 1951, pp. 123-131
AUTHOR'S SUMMARY

Twenty-one laboratory-prepared portland cement clinkers, which contained from 0.03 to 2.0 percent sodium oxide, were examined for reaction and expansion in mortars made with opaline aggregate. Clinkers containing as little as 0.21 percent sodium oxide caused reaction but not expansion. Those containing 0.35 percent or more of sodium oxide caused delayed expansion. Mortars made with slowly-cooled clinker generally expanded more rapidly than those made with quickly-cooled clinker of the same soda content.

Knowledge gained by L. F. E. M. on graded sands for standard test (Experiences acquises par la L. F. E. M. sur les sables gradues pour d'essai normal des liants)

VOELLMY and W. RIMATHE, *Bulletin de la Reunion des Laboratories d'Essais et de Recherches sur les Materiaux et les Constructions*, No. 4, July 1951, pp. 111-157
Reviewed by PHILLIP L. MELVILLE

It was found that the petrographic composition of the sand has usually little influence. The best gradation was between Fuller's and L.F.E.M's. with diameters between 0 and 6 mm. For good control splitting in 3 sizes is necessary with allowable deviations of from ±2 to ± 5 percent. If needed, a partial replacement of natural grains with crushed materials may be considered, specially for diameters smaller than 0.2 mm. The elimination of that size particle decreases workability and increases segregation. Strength properties of cement are best observed with sands between 0 and 6 mm in diameter.

Concrete cracking—Part I, General characteristics

L. BOYD MERCER, *Commonwealth Engineer* (Melbourne), Sept. 1951, pp. 64-69

Author advocates a positive attitude toward cracking of concrete with attempts not only to classify cracks but to detect causes and to apply remedial measures. Before-hardening cracks are attributed to movement of forms and subgrade, settlement of reinforcement or aggregate, and plastic and drying shrinkage of the concrete. Cracks after hardening are classified as due to physical, chemical, and temperature changes, stress concentrations in reinforcement or forms, and to structural design or accident.

Calculation of air bubble size distribution from results of a Rosiwal traverse of aerated concrete

G. W. LORD and T. F. WILLIS, *ASTM Bulletin*, No. 177, Oct. 1951, pp. 56-61
AUTHORS' SUMMARY

Several characteristics of the dispersed air bubbles in aerated concrete have been indicated as important to the study of the function, and intelligent use, of entrained air. This paper describes a method of calculation, together with its graphical derivation, by means of which various parameters characterizing the dispersed air phase may be obtained from measurements made in a modified Rosiwal microscopic traverse. Also included

is an appendix in which the method is derived in more formal mathematical terms. While originally developed for study of concrete, the method is applicable to any system containing spherical dispersoids which can be examined by the Rosiwal linear traverse technique. A sample calculation showing results for a specimen of concrete is included.

A study of combined bending and axial load in reinforced concrete members

EIVIND HOGNESTAD, University of Illinois Engineering Experiment Station, *Bulletin* 399, Urbana, Nov. 1951, 128 pp.

AUTHOR'S SUMMARY

This bulletin presents the methods and results of an experimental and analytical investigation undertaken to throw new light on the behavior and strength of reinforced concrete members subject to combined bending and axial load.

A total of 120 column specimens was tested, 90 of which were 10-in. square tied columns with 1.48 to 4.8 percent reinforcement, and 30 were 12-in. round spiral columns with 4.25 percent longitudinal reinforcement. The concrete quality was varied from 1500 to 5500 psi, and the eccentricity of load varied from 0 to 1.25 times the lateral dimension of the columns.

A critical study of the basic assumptions in inelastic flexural theories for reinforced concrete was made. On the basis of such studies and of phenomena observed in the present tests, a general inelastic flexural theory was developed, by means of which the behavior of the test columns as well as the modes of failure and the ultimate loads could be predicted with good accuracy.

The theory developed by C. S. Whitney and a modification of V. P. Jensen's theory were also compared to the measured ultimate loads, and a satisfactory agreement was found.

The new canal harbor bridge in Heilbronn (Die neue Kanalbruecke in Heilbronn)

W. STOHR, *Beton- u. Stahlbetonbau* (Berlin), V. 45, No. 12, Dec. 1950, pp. 269-274; V. 46, No. 2, Feb. 1951, pp. 30-32

Reviewed by RUDOLPH FISCHL

This bridge is a three-hinged frame with cantilevers and suspended end spans. The main span, 354 ft long, has a rise of 36 ft, and the end spans are 70 ft each. The total width of 49 ft includes two sidewalks, each 6 ft wide.

The frame is formed by two independent box girders with a free supported slab spanned between the two girders, carrying one street car track and serving as horizontal wind bracer.

A special feature of this bridge is the reuse of the old abutments of the former bridge, which was dynamited during World War II. These abutments, though too small for the new bridge, were widened by cantilever construction without the use of new foundations. To avoid tension cracks in the upper corner of the frame, partial prestressing was used.

Structural details, construction and erection problems are discussed and illustrated by many figures and photographs. The bridge was designed and built by Wayss & Freytag A. G.

Recent developments in precasting concrete bridges and structures

R. B. MCMINN, *Bulletin* No. 39, Highway Research Board, 1951, 20 pp.

Discusses recent rapid advances in the use of precast members for building construction and the considerable interest exhibited in applying precasting to bridges and highway structures. One system of precasting combines precast stringers with cast-in-place slabs to form T-beam spans. The second system consists of assembling precast beams and precast slabs and cementing together these structural members. Mention is made of developments in prestressing precast units.

Advantages of precast members include (1) savings in forms, falsework, and placing of concrete; (2) elimination of considerable on-the-job labor; (3) faster construction; (4) facilitation of maintenance of traffic; (5) closer control of concrete mixture, placing, and curing; and (6) expedition of construction through precasting under cover in winter.

The first centennial of reinforced concrete (Il I centenario del cemento armato)

G. NEUMAN, *Il Cemento* (Milan), V. 47, No. 11, Nov. 1950, pp. 186-189

Reviewed by GENNARO MIANULLI

The title is misleading; this is a review of the development and evolution of reinforced concrete construction in Italy during the last fifty years. A brief reference is made to the

contribution by Monier, Cargnet, Hennibique, Freyssinet, Dischinger, Morsch, Wayss, Freytag, and others in France and Germany. Contemporarily, in Italy, Guidi, Danusso, Colonetti, Cavanazzi, Baroni, Poggi, Belluzzi, Morandi and others were busy on theory, development of new structural systems, solving special problems and testing new combinations of materials.

Professor Danusso was the first investigator to observe the behavior of a two-way reinforcement in a slab and to present, in 1911, the first flat plate theory and analysis investigated later by Dischinger and Finsterwalder. At present the interest is so widespread that flat plate is being treated with new mathematical tools and is being simplified for practical application. In the review is included a list of construction firms pioneering in reinforced concrete construction accompanied by a list of bridges and buildings erected in the last fifty-two years.

Research on pumped concrete (Recherches sur le beton pompe)

ALBERT JOISEL, *Cahiers du Centre Scientifique et Technique du Batiment*, V. 11, No. 107 First Quarter, 1951 (also *Annales de l'Institut Technique du Batiment et des Travaux Publics*, No. 204, Sept. 1951)
Reviewed by PHILLIP L. MELVILLE

The coefficient of friction was studied experimentally with a special device. The coefficient was determined for gravel and crushed stone aggregates, for sand and for cements as a function of the ratio moisture or water to material by weights. A considerable amount of data was obtained for mortars and concretes using these materials. Results indicate that friction does not vary appreciably with gradation of the aggregate if the ratio water to material does not change, but is a function of particle shape. The finer the cement, the higher the friction and the more water it is able to retain. Friction in mortars and concretes is a function of the constituents. The information is applied to the pumping of concrete. Since the coefficient of friction first increases with the water-cement ratio, then drops rapidly and tends toward zero, only concrete of proper composition can be pumped. A critical amount of water (equivalent to a slump of 2 to 4 in.) is needed to prevent pressure of the solids against the pipes and to keep the coefficient of friction at less than 2 to 4. Twenty-one references are addended.

Hot air hardening of precast concrete parts, especially of hollow blocks (Heisslufthaertung von Betonfertigteilen insbesondere von Hohlblocksteinen)

A. KLEINLOGEL, *Betonstein-Zeitung* (Wiesbaden), V. 17, No. 1, Jan. 1951, pp. 14-15
Reviewed by RUDOLPH FISCHL

A new patented method which makes it possible to treat precast parts with enough heat to start the binding process within a few minutes, even in winter and during rain, is described. The heat treating process may be repeated as often as necessary to obtain a required degree of hardening sufficient for storage or shipment.

The movable equipment consists of a generator, which blows air mixed with CO_2 and water spray, over a layer of glowing coke and distributes the hot air evenly through a hood over the lined-up material.

Tests made at the Technical University of Darmstadt gave remarkable results. Concrete parts, treated by this process, reached the strength specified by the German code in about half the time as parts without treatment, and they can be stored after 24 hours. After 8 to 10 hours the heat treated parts are safe against damage through frost or rain. The economy of this method is another promising factor.

Prestressed concrete conference stresses precast units

HUBERT C. PERSONS, *Rock Products*, V. 54, No. 10, Oct. 1951, pp. 174-178 and 186
Reviewed by DONALD M. AGRIMSON

A summary of what was discussed at the First United States Conference on Prestressed Concrete.

This conference, held at the Massachusetts Institute of Technology, August 1951, emphasized economy and quality of precast units and stressed large volume market for development by concrete products industry. There was general agreement that a vast amount of testing and research must be done so that structural designers may become as familiar with prestressed concrete as they are with reinforced concrete.

Discussions at the conference were conducted under three main categories: (a) various special applications of prestressed concrete, including tanks, pipe, bridges, buildings, pavement, piles, and mass produced units, (b) materials, including concrete, wire, and bars, and (c) design and re-

search, including general considerations of design, basic concepts of design, design of continuous beams, a suggestion for the urgency of determining the characteristics of prestressed concrete under adverse weather conditions, a description of load tests on a railroad trestle slab, and a paper describing wire tests and anchorage studies on experimental slabs.

Tests of prestressed concrete beams with alloy steel bars

A. D. Ross, *Magazine of Concrete Research* (London), No. 7, Aug. 1951, pp. 9-18

From Author's Summary

Reports a series of tests of beams prestressed by a system developed by Donovan Lee which utilizes alloy steel bars with an ultimate tensile strength of about 140,000 psi in diameters up to 1⅛ in. Details are given of the five I- and U-shaped test beams. Some were assembled from precast units and others were monolithic, spans varied from 21 ft 6 in. to 46 ft 6 in. and the total prestressing force from 27.5 to 89.1 tons.

The method of loading, sequence of the tests and the recording of deflections and strains are described. Typical strain and deflection curves are given, and the results are presented in tables showing loads at cracking and failure, deflections at working loads and ultimate load factors.

Measurements on the steel were obtained in one of the beams, and data on the strain during prestressing, anchoring and loading are presented.

Freezing and thawing tests of concrete containing Oregon and Washington aggregates

A. G. Timms, *Public Roads*, Oct. 1951, pp. 206-216

Author's Summary

In this study, plain and air-entraining concretes made with certain Oregon and Washington aggregates, and with Potomac River aggregates as a control, were subjected to alternate freezing and thawing after preliminary moist curing of both short and prolonged periods. Some of the concretes had poor resistance to freezing and thawing while others stood up well. There was no evidence in most cases of any alkali-aggregate reaction.

Prolonged initial curing improved the resistance to freezing and thawing of all but one of the non-air-entraining concretes. This was not true for the air-entraining concretes, and in many cases the additional curing was actually harmful. However, the air-entraining concretes with prolonged initial curing generally had better resistance to freezing and thawing than the comparable non-air-entraining concretes cured only for a short time.

In all cases, when the preliminary curing was short, the use of an air-entraining agent produced concrete with greater resistance to freezing and thawing. With the prolonged initial curing, air entrainment was not beneficial to three of the five aggregates.

In general, the tests indicated that the durability of air-entraining concrete may be adversely affected by conditions which involve long-continued exposure to moisture. It was also found that the results of freezing and thawing tests made after a short period of curing in moist air may not be sufficient to evaluate properly the durability of concrete containing certain cement-aggregate combinations.

Principles underlying the steam curing of concrete at atmospheric pressure

A. G. A. Saul, *Magazine of Concrete Research* (London), No. 6, Mar. 1951, pp. 127-140

From Author's Summary

Summarizes conclusions drawn from experimental studies concerning the principles underlying steam curing at atmospheric pressure. It is shown that if the temperature gradient of the concrete after mixing does not exceed a certain value, the concrete gains strength during and after treatment in relation to its maturity (reckoned by temperature-time) approximately in accordance with the same law as holds for normally cured concrete. Concrete which is raised in temperature more rapidly is shown not to obey this law, and to be adversely affected in strength at a later age.

The use of the too-rapid early temperature rises often employed in practice introduces various opposing variables which suggest optimum temperatures, delayed treatments and other arrangements of the curing cycles. Such expedients are unnecessary, however, if a slow initial temperature gradient is used.

Title No. 48-30

Analysis of Skewed Rigid Frames and Arches*

By JAMES P. MICHALOS†

SYNOPSIS

A numerical procedure for the analysis of single-span skewed rigid frames and arches, subjected to loads or deformations in any direction, is presented. The structure may have any shape and any variation in cross section along its length. Several examples are included.

INTRODUCTION

Modern highway requirements in general make unacceptable the use of awkward alignment to eliminate skew crossings. Many such crossings can be advantageously bridged by skewed rigid frames or arches of concrete. A solution for the reactions of a skew arch, involving six rather involved simultaneous equations, was introduced by J. Charles Rathbun.‡ In the attempt to make Rathbun's analysis more understandable and easier to apply, a number of men have proposed certain simplifications. Rathbun§ and Weiner** dropped certain terms which they considered of secondary importance. Hodges†† and Barron‡‡ have obtained results by comparison of skewed frames with right frames.

This paper presents a general procedure for the analysis of skewed rigid frames and arches. The structure considered may be of any shape with any variation in cross section along its length, and loads or deformations may be applied in any direction. A complicated analysis of a space structure is reduced to a mechanical procedure involving only arithmetic. The results, however, are identical with those obtained by use of a classical method, such as used by Rathbun.§§ They are exact if the relationships between the statics and geometry of an element of the structure, as presented herein and implied or stated by Rathbun and others, are considered acceptable. Total moments and shears are computed, and moment diagrams are drawn. Numerical checks of the geometry are included. The distribution of the moments and shears in a cross section is not discussed in this paper.

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†Member American Concrete Institute, Assoc. Professor of Civil Engineering, Iowa State College, Ames.

‡Rathbun, J. Charles, "Analysis of Stresses in the Ring of a Skew Arch," *Transactions*, ASCE, V. 87, 1924, p. 611.

§*Ibid.*

**Weiner, Bernard L., "Design of a Reinforced Concrete Skew Arch," *Transactions*, ASCE, V. 96, 1932, p. 1212.

††Hodges, Richard M., "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," *Transactions*, ASCE, V. 109, 1944, p. 913.

‡‡Barron, Maurice, "Reinforced Concrete Skewed Rigid-Frame and Arch Bridges," *Transactions*, ASCE, V. 116, Apr. 1951, p. 999.

§§Rathbun, *op. cit.*

GENERAL PROCEDURE FOR STRUCTURES IN SPACE

The correct analysis of any statically indeterminate structure simultaneously satisfies the requirements of statics and geometry. To satisfy the requirements of statics, the sum of the forces in any direction and the sum of the products of forces times distances (moments) about any axis, must be equal to zero. The requirements of geometry are such that for a closed circuit the sum of the angle changes about any axis and the sum of the products of angle changes and distances (displacements), must equal zero. It follows that an analogy can be drawn between angle changes and forces, and between displacements and moments; and this analogy between computations of geometry and of statics forms a basis for the method of analysis to be presented.

A two-step procedure has been presented elsewhere* for the analysis of curved or segmental structures continuous in space between two supports. That procedure can be stated as follows:

1. Assume any statically possible distribution of moments along the structure. In general such an assumption will introduce errors in the geometry of the deflected structure.

2. Add (or subtract) a distribution of moments which corrects the errors in geometry without disturbing statical equilibrium.

Such a procedure will automatically satisfy the requirements of statics.

In the following section the detailed analytical procedure for application to skewed arches and rigid frames is developed. It is based on the general procedure for structures curved in space.*

ANALYTICAL PROCEDURE FOR SKEWED RIGID FRAMES AND ARCHES

All moments and forces, and all angle changes and displacements, are related to a system of rectangular axes as shown in Fig. 1. Forces, moments, rotations and displacements acting on a cross section are represented as vectors. They are considered positive on a front face when they are in the direction of an axis, and positive on a back face when they are against the direction of an axis. Use the right-hand rule for sense of moments. The effects of shear deformation are omitted, although it is possible to include them in the analysis.

Using a two-step procedure, as previously outlined, the final total moments and forces at any section of the structure will be obtained as follows:

$$\left.\begin{array}{lll} m_x = m_{xo} - m_{xi}; & m_y = m_{yo} - m_{yi}; & m_z = m_{zo} - m_{zi} \\ v_x = v_{xo} - v_{xi}; & v_y = v_{yo} - v_{yi}; & v_z = v_{zo} - v_{zi} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where the subscripts o and i refer to the assumed distribution and the correction distribution, respectively.

In Fig. 2a three particular moments and forces are projected in each orthogonal plane. The subscript c indicates that the forces are along axes which shall be termed centroidal axes. The positions of these axes, which are determined by six bar distances, for example $\bar{x}_y$, will be defined later. In each

*Baron, Frank and Michalos, James P., "Laterally Loaded Plane Structures and Structures Curved in Space," *Proceedings*, ASCE, V. 76, Separate No. 51, Jan. 1951.

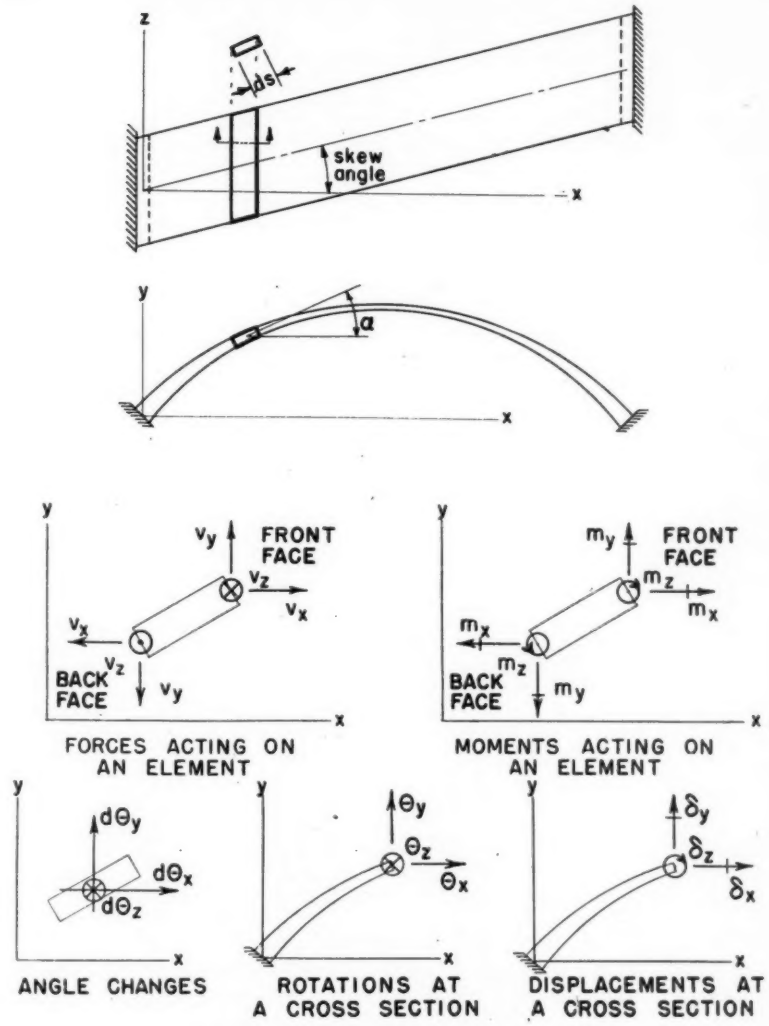
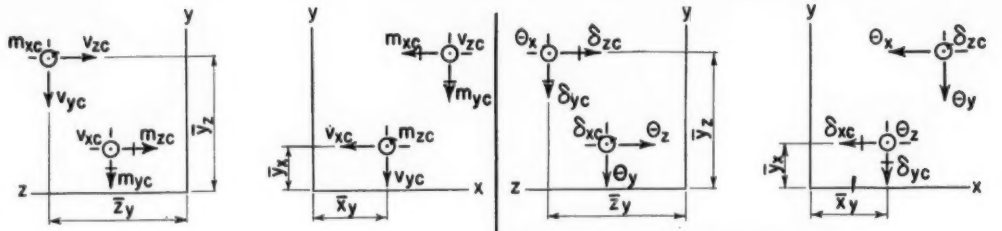
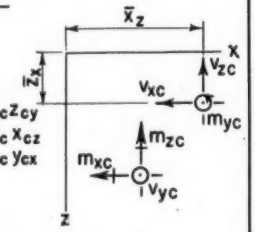


Fig. 1—Reference system and sign convention



CORRECTION SHEARS AND MOMENTS AT ANY SECTION OF THE STRUCTURE :-

$$v_{xi} = v_{xc}$$
$$v_{yi} = v_{yc}$$
$$v_{zi} = v_{zc}$$
$$m_{xi} = m_{xc} + v_{zc}y_{cz} - v_{yc}z_{cy}$$
$$m_{yi} = m_{yc} + v_{xc}z_{cx} - v_{zc}x_{cz}$$
$$m_{zi} = m_{zc} + v_{yc}x_{cy} - v_{xc}y_{cx}$$

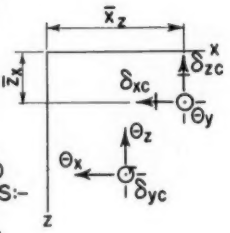


NOTE: Terms such as y_{cz} represent centroidal distances to a section of the structure.

(a) STATICAL RELATIONSHIPS

ROTATIONS AND DISPLACEMENTS DUE TO ASSUMED MOMENTS :-

$$\sum d\theta_{xo} = \sum m_{xo}da_{xx} + \sum m_{yo}da_{xy}$$
$$\sum d\theta_{yo} = \sum m_{yo}da_{yy} + \sum m_{xo}da_{yx}$$
$$\sum d\theta_{zo} = \sum m_{zo}da_{zz}$$
$$\delta_{xco} = \sum z_{cx}d\theta_{yo} - \sum y_{cx}d\theta_{zo}$$
$$\delta_{yco} = \sum x_{cy}d\theta_{zo} - \sum z_{cy}d\theta_{xo}$$
$$\delta_{zco} = \sum y_{cz}d\theta_{xo} - \sum x_{cz}d\theta_{yo}$$



ROTATIONS AND DISPLACEMENTS DUE TO CORRECTION MOMENTS:-

$$\sum d\theta_{xi} = \sum m_{xi}\,da_{xx} + \sum m_{yi}\,da_{xy}$$
$$\sum d\theta_{yi} = \sum m_{yi}\,da_{yy} + \sum m_{xi}\,da_{yx}$$
$$\sum d\theta_{zi} = \sum m_{zi}\,da_{zz}$$
$$\delta_{xci} = \sum z_{cx}\,d\theta_{yi} - \sum y_{cx}\,d\theta_{zi}$$
$$\delta_{yci} = \sum x_{cy}\,d\theta_{zi} - \sum z_{cy}\,d\theta_{xi}$$
$$\delta_{zci} = \sum y_{cz}\,d\theta_{xi} - \sum x_{cz}\,d\theta_{yi}$$

(b) GEOMETRICAL RELATIONSHIPS

Fig. 2—Statical and geometrical relationships with respect to centroidal axes

orthogonal plane the projections of two moments and a force have been grouped as in the shear and torsion analogy,* and the projections of two forces and a moment have been grouped as in the column analogy.† Also shown in Fig. 2a are expressions relating the correction moments and forces, m_i and v_i, at any section of the structure to the correction moments and forces, m_c and v_c, associated with the centroids.

Rotations, Θ, about x, y and z axes, and displacements, δ_c, along the centroidal axes are grouped in Fig. 2b. Axes of rotation correspond to those about which moments are applied in Fig. 2a, and axes of displacement correspond to those along which forces are applied in Fig. 2a. Included in Fig. 2b are expressions for rotations and displacements, along centroidal axes, due to assumed moments and due to correction moments. Observe that moments about an x axis or a y axis contribute to angle changes about both axes. In the expressions of Fig. 2b

$$da_{xx} = ds/K_{xx}; \quad da_{yy} = ds/K_{yy}; \quad da_{zz} = ds/K_{zz}; \quad da_{xy} = ds/K_{xy}; \quad da_{yx} = ds/K_{yx}. \tag{2}$$

The da terms represent angle changes due to unit moments. For example, da_{xx} and da_{xy} represent angle changes, in an element ds, about an x axis due to unit moments applied about the x axis and y axis, respectively. The K terms are stiffness factors, and represent moments required to produce unit angle change. For example, K_{xx} and K_{xy} represent moments about an x axis and y axis, respectively, necessary to produce unit angle change about an x axis. It can be shown that K_{yx} is equal to K_{xy}. From previously published‡ general expressions for such stiffness factors, the following relations for an element of a skewed rigid frame or skewed arch were obtained:

$$\left.\begin{aligned} K_{xx} &= \frac{K_{n1}\,K_t}{K_{n1}\cos^2\alpha + K_t\sin^2\alpha}; \quad K_{yy} = \frac{K_{n1}\,K_t}{K_{n1}\sin^2\alpha + K_t\cos^2\alpha} \\ K_{zz} &= K_{n2}; \quad K_{xy} = \frac{K_{n1}\,K_t}{(K_{n1} - K_t)\sin\alpha\cos\alpha} \end{aligned}\right\} \tag{3}$$

where

$$K_{n1} = EI_1; \quad K_{n2} = EI_2; \quad K_t = GJ$$

and where

α = slope of the element projected on the xy plane (see Fig. 1).
E = modulus of elasticity as generally defined.
G = modulus of rigidity, or modulus of elasticity in shear.
I_1 = moment of inertia of a cross section about the section's major principle axis.
I_2 = moment of inertia of a cross section about the section's minor principle axis.
J = a torsional factor such that GJ is the moment, about the axis of twist, necessary to deform a segment a unit angle per unit length about the same axis.

Geometric requirements

Now consider the requirements of geometry. Since the sum of the angle changes and the sum of the products of angle changes and corresponding distances must both equal zero, it follows that the angle changes and displacements due to the correction moments must balance those due to the

*Ibid.
†Cross, Hardy, "The Column Analogy," *Bulletin 215*, Eng. Exp. Sta., University of Illinois, Urbana, Ill., 1930.
‡Baron and Michalos, *op. cit.*

assumed moments, and we can write:

$$\left.\begin{aligned} \Sigma d\Theta_x &= \Sigma d\Theta_{xo} - \Sigma d\Theta_{xi} = 0; \quad \Sigma d\Theta_y = \Sigma d\Theta_{yo} - \Sigma d\Theta_{yi} = 0; \\ \Sigma d\Theta_z &= \Sigma d\Theta_{zo} - \Sigma d\Theta_{zi} = 0 \end{aligned}\right\} \quad (4)$$

and

$$\delta_x = \delta_{xco} - \delta_{xci} = 0; \quad \delta_y = \delta_{yco} - \delta_{yci} = 0; \quad \delta_z = \delta_{zco} - \delta_{zci} = 0 \quad (5)$$

From the relationships in Fig. 2a and Fig. 2b, Eqs. (4) can be written as follows:

$$\left.\begin{aligned} \Sigma d\Theta_x &= \Sigma m_{xo} da_{xx} + \Sigma m_{yo} da_{xy} - m_{xc}\Sigma da_{xx} - v_{zc}\Sigma y_{cz} da_{xx} + v_{yc}\Sigma z_{cy} da_{xx} \\ &\quad - m_{yc}\Sigma da_{xy} - v_{xc}\Sigma z_{cx} da_{xy} + v_{zc}\Sigma x_{cz} da_{xy} = 0 \\ \Sigma d\Theta_y &= \Sigma m_{yo} da_{yy} + \Sigma m_{xo} da_{yx} - m_{yc}\Sigma da_{yy} - v_{xc}\Sigma z_{cx} da_{yy} + v_{zc}\Sigma x_{cz} da_{yy} \\ &\quad - m_{xc}\Sigma da_{yx} - v_{zc}\Sigma y_{cx} da_{yx} + v_{yc}\Sigma z_{cy} da_{yx} = 0 \\ \Sigma d\Theta_z &= \Sigma m_{zo} da_{zz} - m_{zc}\Sigma da_{zz} - v_{yc}\Sigma x_{cy} da_{zz} + v_{xc}\Sigma y_{cx} da_{zz} = 0 \end{aligned}\right\} \quad (6)$$

If centroidal axes are chosen such that

$$\left.\begin{aligned} \Sigma x_{cz} da_{xy} - \Sigma y_{cz} da_{xx} &= 0 \qquad \Sigma z_{cx} da_{yy} = 0 \\ \Sigma z_{cy} da_{xx} &= 0 \qquad \Sigma x_{cy} da_{zz} = 0 \\ \Sigma x_{cz} da_{yy} - \Sigma y_{cz} da_{yx} &= 0 \qquad \Sigma y_{cx} da_{zz} = 0 \end{aligned}\right\} \quad (7)$$

then v_{zc} and v_{yc} will not contribute to rotation about the x axis, v_{zc} and v_{xc} will not contribute to rotation about the y axis, and v_{yc} and v_{xc} will not contribute to rotation about the z axis. Eqs. (6) then become

$$\left.\begin{aligned} \Sigma d\Theta_x &= \Sigma m_{xo} da_{xx} + \Sigma m_{yo} da_{xy} - m_{xc}\Sigma da_{xx} - m_{yc}\Sigma da_{xy} - v_{xc}\Sigma z_{cx} da_{xy} = 0 \\ \Sigma d\Theta_y &= \Sigma m_{yo} da_{yy} + \Sigma m_{xo} da_{yx} - m_{yc}\Sigma da_{yy} - m_{xc}\Sigma da_{yx} + v_{yc}\Sigma z_{cy} da_{yx} = 0 \\ \Sigma d\Theta_z &= \Sigma m_{zo} da_{zz} - m_{zc}\Sigma da_{zz} = 0 \end{aligned}\right\} \quad (8)$$

Also from the relationships shown in Fig. 2a and Fig. 2b, and in addition using Eqs. (7), Eqs. (5) can be written as follows:

$$\begin{aligned} \delta_x &= \Sigma z_{cx}(m_{yo} da_{yy} + m_{xo} da_{yx}) - \Sigma y_{cx} m_{zo} da_{zz} - v_{xc}\,(\Sigma z^2{}_{cx} da_{yy} + \Sigma y^2 da_{zz}) \\ &\quad - v_{yc}\,(-\Sigma z_{cx} z_{cy} da_{yx} - \Sigma x_{cy} y_{cx} da_{zz}) - v_{zc}\,(\Sigma y_{cz} z_{cx} da_{yx} - \Sigma x_{cz} z_{cx} da_{yy}) \\ &\quad - m_{zc}\Sigma z_{cx} da_{yx} = 0 \\ \delta_y &= \Sigma x_{cy} m_{zo} da_{zz} - \Sigma z_{cy}(m_{xo} da_{xx} + m_{yo} da_{xy}) - v_{xc}\,(-\Sigma z_{cx} z_{cy} da_{xy} \\ &\quad - \Sigma x_{cy} y_{cx} da_{zz}) - v_{yc}\,(\Sigma x^2{}_{cy} da_{zz} + \Sigma z^2{}_{cy} da_{xz}) \\ &\quad - v_{zc}\,(\Sigma z_{cy} x_{cz} da_{xy} - \Sigma z_{cy} y_{cx} da_{xz}) - m_{yc}\Sigma z_{cy} da_{xy} = 0 \\ \delta_z &= \Sigma y_{cz}(m_{xo} da_{xx} + m_{yo} da_{xy}) - \Sigma x_{cz}(m_{xo} da_{yx} + m_{yo} da_{yy}) \\ &\quad - v_{xc}\,(\Sigma y_{cz} z_{cx} da_{xy} - \Sigma x_{cz} z_{cx} da_{yy}) - v_{yc}\,(\Sigma z_{cy} x_{cz} da_{yx} - \Sigma z_{cy} y_{cx} da_{xx}) \\ &\quad - v_{zc}\,(\Sigma y^2{}_{cz} da_{xx} - 2\Sigma x_{cz} y_{cx} da_{xy} + \Sigma x^2{}_{cz} da_{yy}) = 0 \end{aligned} \quad (9)$$

From Eqs. (7) it is possible to write expressions for the bar distances, which locate the centroidal axes with respect to an arbitrary set of reference axes, such that

$$\left.\begin{aligned} x_{yc} &= x - \bar{x}_y; \quad x_{zc} = x - \bar{x}_z; \quad y_{zc} = y - \bar{y}_z \\ y_{xc} &= y - \bar{y}_x; \quad z_{xc} = z - \bar{z}_x; \quad z_{yc} = z - \bar{z}_y \end{aligned}\right\} \quad (10)$$

The expressions for the bar distances are summarized in Fig. 3.

Returning to Eqs. (8) and (9), these can be rewritten

$$\left.\begin{aligned} \Sigma d\Theta_x &= A_x m_{xc} + A_{xy} m_{yc} + f_{xx} v_{xc} - P_x = 0 \\ \Sigma d\Theta_y &= A_{yx} m_{xc} + A_y m_{yc} + f_{yy} v_{yc} - P_y = 0 \\ \Sigma d\Theta_z &= A_z m_{zc} - P_z = 0 \end{aligned}\right\} \quad (11)$$

and

$$\left.\begin{aligned} \delta_x &= t_{xx} m_{xc} + J_x v_{xc} + J_{xy} v_{yc} + J_{xz} v_{zc} - M_x = 0 \\ \delta_y &= t_{yy} m_{yc} + J_{yx} v_{xc} + J_y v_{yc} + J_{yz} v_{zc} - M_y = 0 \\ \delta_z &= J_{zx} v_{xc} + J_{zy} v_{yc} + J_z z_{zc} - M_z = 0 \end{aligned}\right\} \quad (12)$$

where the coefficients and the P and M terms are as shown in Fig. 3. The coefficients represent total rotations and displacements due to unit moments and unit forces associated with the centroidal axes. For example, A_{xy} is equal to $\Sigma\, da_{xy}$, and represents a rotation about an x axis due to a moment m_{yc} equal to one. The coefficient J_{xy} is equal to $(-\Sigma\, z_{cx}z_{cy}da_{xy} - \Sigma\, x_{cy}y_{cx}da_{xz})$. This coefficient represents a displacement along the centroidal axis associated with v_{xc}, and is due to a force v_{yc} equal to one. Since K_{yx} is equal to K_{xy}, da_{yx} is equal to da_{xy}. Consequently A_{yx} will be numerically equal to A_{xy} and J_{yx} will be numerically equal to J_{xy}. The coefficient f_{xx} is equal to $\Sigma\, z_{cx}da_{xy}$, and represents a rotation about an x axis due to a force v_{xc} equal to one. The coefficient t_{xx} is equal to $\Sigma\, z_{cx}da_{yx}$, and will therefore be numerically equal to f_{xx}. The former represents a displacement along the centroidal axis associated with v_{xc}, and is due to a moment m_{xc} equal to one.

The P and M terms represent rotations and displacements, respectively, and are due to the assumed distributions of moment. For example, P_x represents the total rotation about an x axis, and M_x represents a displacement in the x direction along the centroidal axis. If a structure is analyzed for the effects of volume change, the P terms are equal to zero and the M terms are equal to the displacements which would occur if the structure were free to move at the supports.

Because of the analogy that can be drawn between angle changes and forces, and between displacements and moments, computations of geometry can be handled as computations of statics. This is done by treating angle changes and rotations as forces along the axes of rotation, and treating displacements as moments about the axes of displacement. As a result of this concept, involving interchange of computations, and from the relations in Fig. 2a, the solution of the equations of geometry (Eqs. 11 and Eqs. 12) can be written in the following form:

$$\left.\begin{aligned} v_{xi} &= \frac{\mathcal{M}_x'}{J_x'} & m_{xi} &= \frac{\mathcal{P}_x'}{\mathcal{A}_x'} + \frac{\mathcal{M}_z' y_{cz}}{J_z'} - \frac{\mathcal{M}_y' z_{cy}}{J_y'} \\ v_{yi} &= \frac{\mathcal{M}_y'}{J_y'} & m_{yi} &= \frac{\mathcal{P}_y'}{\mathcal{A}_y'} + \frac{\mathcal{M}_x' z_{cx}}{J_x'} - \frac{\mathcal{M}_z' x_{cz}}{J_z'} \\ v_{zi} &= \frac{\mathcal{M}_z'}{J_z'} & m_{zi} &= \frac{\mathcal{P}_z'}{\mathcal{A}_z'} + \frac{\mathcal{M}_y' x_{cy}}{J_y'} - \frac{\mathcal{M}_x' y_{cx}}{J_x'} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

In these equations the $\mathcal{P}'$ terms represent rotations due to assumed distributions of moment, and are the total analogous forces. The $\mathcal{M}'$ terms represent displacements along centroidal axes, due to the assumed distributions of moment, and are the total analogous moments. The $\mathcal{A}'$ terms and the J' terms represent rotations and displacements, respectively, resulting from unit moments and forces applied along centroidal axes, and may be considered as analogous areas and moments of inertia.

All terms involved in the procedure are summarized in Fig. 3, which is a tabular form for use in determining moments and forces in a skewed rigid frame or arch. Fig. 4 consists of a simplified tabular form which results when a yz-plane can divide the structure into two equivalent parts. Such a

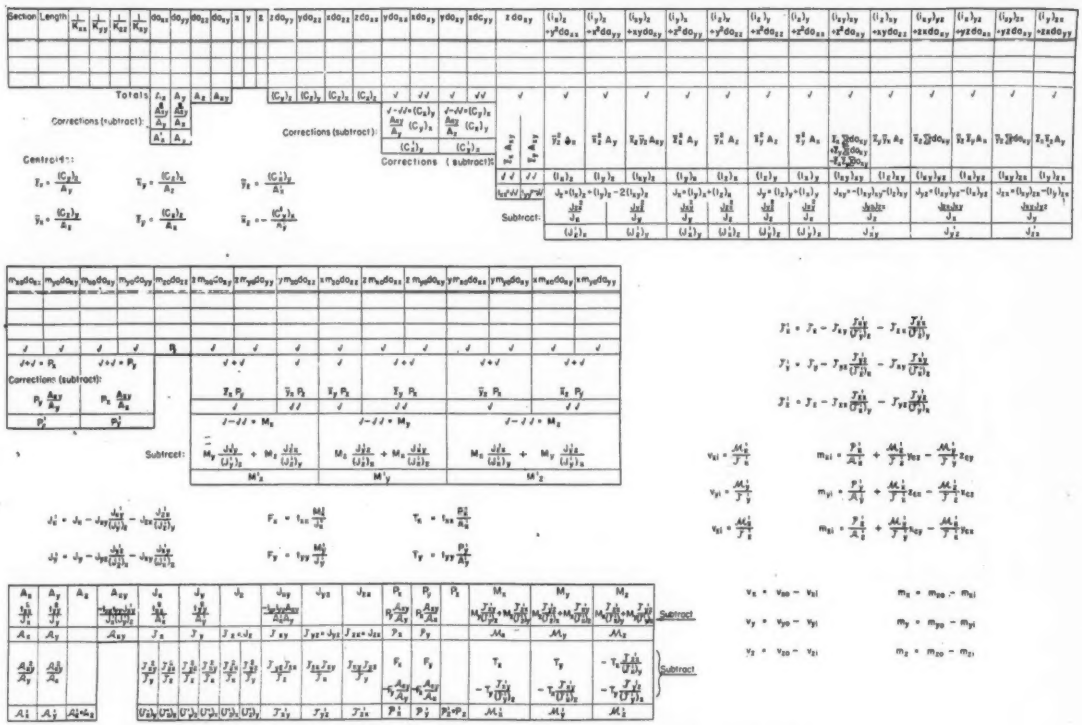


Fig. 3—General procedure for analysis of skewed frames and arches

skewed frame or arch is referred to herein as a symmetrical structure, and it is necessary to compute the properties of only one-half of it and then double the sums. However, the signs of K_{xy} are opposite in the two halves of the structure, and this should be kept in mind in computing values of $m_{xo}da_{xy}$ and $m_{yo}da_{xy}$. In the general case any convenient set of rectangular axes may be chosen, with the exception that, in a structure hinged about z axes at the supports, the x axis of reference should pass through the hinges. For a symmetrical structure, as defined above, the y axis must be an axis of symmetry if advantage is to be taken of the simplified computation schedule in Fig. 4. In making numerical computations, the structure is divided into segments of finite length, and the elastic areas, da, are those of the finite segment. As a result, in computing the analogous moments of inertia and products of inertia, such as $(I_x)_z$ and $(I_{xy})_z$, it may be necessary to add corrections, $(i_x)_z$ and $(i_{xy})_z$ to y^2da_{xx} and $xyda_{xy}$, respectively. These corrections are moments of inertia and products of inertia of the elastic areas about their own centroidal axes.

NUMERICAL EXAMPLES

Constant cross section

The analysis of a hinged, skewed rigid frame, of constant cross section, is shown in Fig. 5. The deck supports a load of 0.1 kip per sq ft. The assumed curves of moment are consistent with the structure considered simply supported (one end on rollers) at ends A and D. Values of the bending stiffnesses, $K_{n1} = EI_1$ and $K_{n2} = EI_2$, and the torsional stiffness, $K_t = GJ$, are given in the figure. Although the frame is symmetrical, it was considered preferable for present purposes not to take advantage of this. Attention is called to the fact that, in computing the analogous moments of inertia, corrections were added to such terms as y^2da_{xx}. For example, the contribution of member AB to $(I_x)_z$ is equal to

$$\begin{aligned}(i_x)_z + y^2da_{xx} &= 1/12\ (da_{xx})\ (\text{length})^2 + y^2da_{xx} \\ &= 1/12\ (0.42 \times 10^{-10})\ (12)^2 + (6)^2\ (0.42 \times 10^{-10}) \\ &= 0.020\ (10)^{-7}\end{aligned}$$

The final moments at various sections were obtained by subtracting the computed correction moments from the corresponding assumed moments, and the final moment curves were then drawn.

A check of the geometry is made in the lower right-hand corner of Fig. 5. In general, it is necessary to satisfy the following conditions of geometry:

$$\left.\begin{array}{llll} \Sigma d\Theta_x = \Sigma m_x da_{xx} + \Sigma m_y da_{xy} = 0 & & \delta_x = \Sigma z_{cz} d\Theta_y - \Sigma y_{cz} d\Theta_z = 0 \\ \Sigma d\Theta_y = \Sigma m_y da_{yy} + \Sigma m_x da_{xy} = 0 & & \delta_y = \Sigma x_{cy} d\Theta_z - \Sigma z_{cy} d\Theta_x = 0 \\ \Sigma d\Theta_z = \Sigma m_z da_{zz} = 0 & & \delta_z = \Sigma y_{cz} d\Theta_x - \Sigma x_{cz} d\Theta_y = 0 \end{array}\right\} \quad \ldots\ldots\ldots (14)$$

In a hinged structure, such as that analyzed in Fig. 5, the conditions $\Sigma\, d\Theta_z = 0$, and $\delta_y = 0$ are not applicable. An inspection of the numerical check indicates a virtually exact fulfillment of the requirements of geometry.

Certain tabulated values in Fig. 5 can be eliminated with only slight loss in accuracy. These are the relatively small values of da_{xx} and da_{yy} which are dependent upon the large bending stiffness, K_{n2}. As might be expected, because of this very large bending resistance, the elimination of all computations

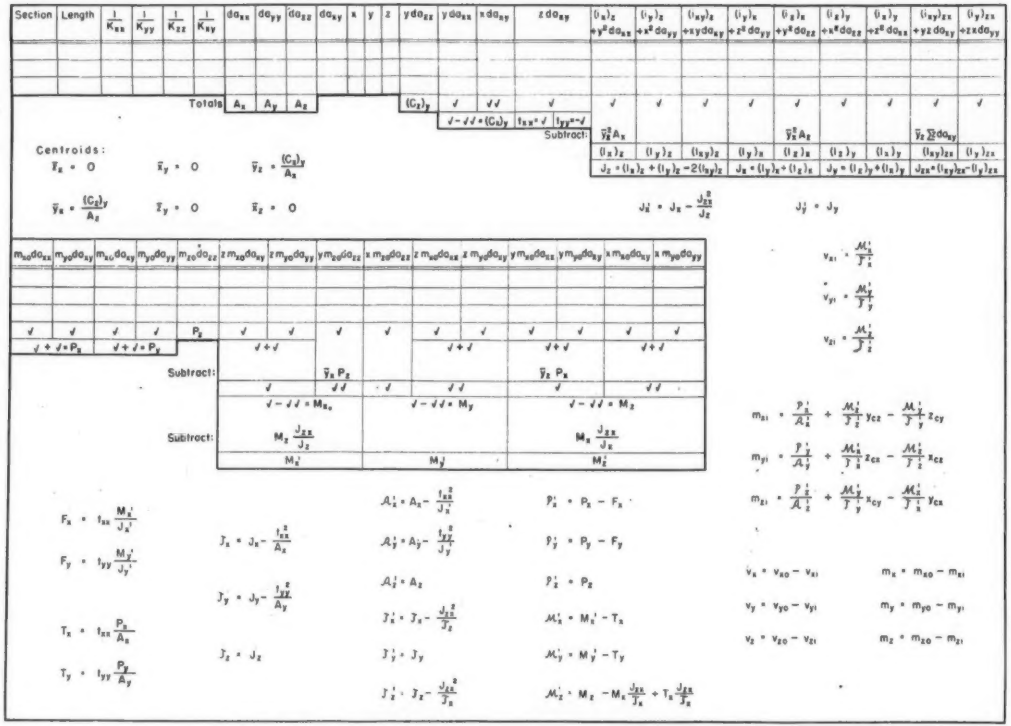
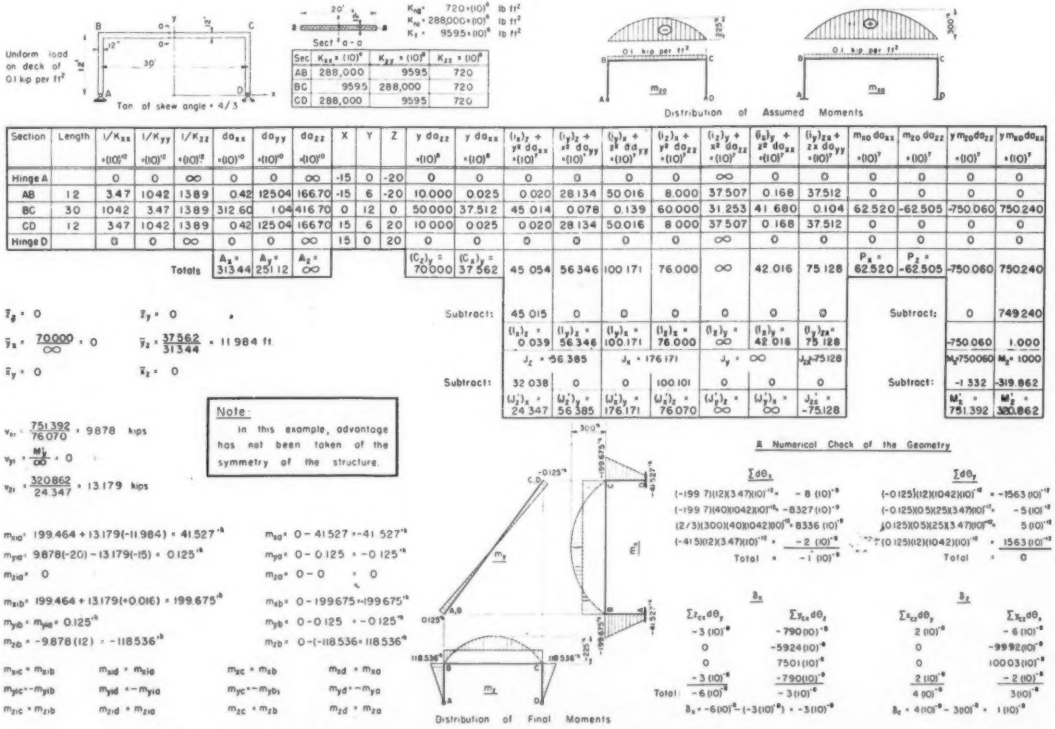


Fig. 4—Procedure for analysis of symmetrical skewed frames and arches



Section	Length	$1/K_{xx}$ ×(10)⁻¹²	$1/K_{yy}$ ×(10)⁻¹²	$1/K_{zz}$ ×(10)⁻¹²	da_{xx} ×(10)⁻¹²	da_{yy} ×(10)⁻⁶	da_{zz} ×(10)⁻⁶	X	Y	Z	$y\,da_{zz}$ ×(10)⁻⁶	$y\,da_{xx}$ ×(10)⁻⁶	$(i_x)_z + y^2\,da_{xx}$ ×(10)⁻⁶	$(i_y)_z + x^2\,da_{yy}$ ×(10)⁻⁶	$(i_y)_x + z^2\,da_{yy}$ ×(10)⁻⁶	$(i_z)_x + y^2\,da_{zz}$ ×(10)⁻⁶	$(i_z)_y + x^2\,da_{zz}$ ×(10)⁻⁶	$(\theta_x)_y + z^2\,da_{xx}$ ×(10)⁻⁶	$(i_y)_{zx} + zx\,da_{yy}$ ×(10)⁻⁶	$m_{xo}\,da_{xx}$ ×(10)⁻⁶	$m_{zo}\,da_{zz}$ ×(10)⁻⁶	$y\,m_{zo}da_{zz}$ ×(10)⁻⁶	$y\,m_{xo}da_{xx}$ ×(10)⁻⁶
Hinge A		0	0	∞	0	0	∞	-15	0	-20	0	0	0	0	0	0	∞	0	0	0	0	0	0
AB	12	347	1042	1389	042	12504	166.70	-15	6	-20	10.000	0.025	0.020	28.134	50.016	8.000	37.507	0.168	37.512	0	0	0	0
BC	30	1042	347	1389	312.60	104	416.70	0	12	0	50.000	37.512	45.014	0.078	0.139	60.000	31.253	41.680	0.104	62.520	-62.505	-750.060	750.240
CD	12	347	1042	1389	042	12504	166.70	15	6	20	10.000	0.025	0.020	28.134	50.016	8.000	37.507	0.168	37.512	0	0	0	0
Hinge D		0	0	∞	0	0	∞	15	0	20	0	0	0	0	0	0	0	0	0	0	0	0	0
Totals					A_x = 313.44	A_y = 251.12	A_z = ∞				$(C_z)_y$ = 70.000	$(C_z)_x$ = 37.562	45.054	56.346	100.171	76.000	∞	42.016	75.128	P_x = 62.520	P_z = -62.505	-750.060	750.240
Subtract:													45.015	0	0	0	0	0	0	Subtract:		0	749.240
													$(i_x)_z$ = 0.039	$(i_y)_z$ = 56.346	$(i_y)_x$ = 100.171	$(i_z)_x$ = 76.000	$(i_z)_y$ = ∞	$(i_x)_y$ = 42.016	$(i_y)_{zx}$ = 75.128			-750.060	1.000
													J_z = 56.385		J_x = 176.171		J_y = ∞		J_{zx} 75.128			M_y 750060	M_x = 1000
Subtract:													32.038	0	0	100.101	0	0	0	Subtract:		-1.332	-319.862
													$(J_z')_x$ = 24.347	$(J_z')_y$ = 56.385	$(J_y')_y$ = 176.171	$(J_z')_x$ = 76.070	$(J_y')_z$ = ∞	$(J_x')_z$ = ∞	J_{zx}' = -75.128			M_z' = 751.392	M_x' = 320.862

$\bar{x}_g$ = 0 $\quad \bar{z}_y$ = 0

$\bar{y}_x = \frac{70.000}{\infty} = 0$ $\quad \bar{y}_z = \frac{37.562}{31.344} = 11.984$ ft

$\bar{z}_y$ = 0 $\quad \bar{z}_x$ = 0

Note:
In this example, advantage has not been taken of the symmetry of the structure.

$v_{z_1} = \frac{751.392}{76.070} = 9.878$ kips

$v_{y_1} = \frac{M_z'}{\infty} = 0$

$v_{z_2} = \frac{320.862}{24.347} = 13.179$ kips

$m_{xa} = 199.464 + 13.179(-11.984) = 41.527$ ft-k $\quad m_{xd} = 0 - 41.527 = -41.527$ ft-k

$m_{ya} = 9.878(-20) - 13.179(-15) = 0.125$ ft-k $\quad m_{yd} = 0 - 0.125 = -0.125$ ft-k

$m_{za} = 0$ $\quad m_{zd} = 0 - 0 = 0$

$m_{xb} = 199.464 + 13.179(+0.016) = 199.675$ ft-k $\quad m_{xb} = 0 - 199.675 = -199.675$ ft-k

$m_{yb} = m_{ya} = 0.125$ ft-k $\quad m_{yb} = 0 - 0.125 = -0.125$ ft-k

$m_{zb} = -9.878(12) = -118.536$ ft-k $\quad m_{zb} = 0 - (-118.536) = 118.536$ ft-k

$m_{xc} = m_{xb}$ $\quad m_{xd} = m_{xa}$ $\quad m_{yc} = m_{zb}$ $\quad m_{zd} = m_{za}$

$m_{yc} = -m_{yb}$ $\quad m_{yd} = -m_{ya}$ $\quad m_{zc} = m_{zb}$ $\quad m_{yd} = m_{ya}$

$m_{zc} = m_{zb}$ $\quad m_{zd} = m_{za}$

A Numerical Check of the Geometry

$\Sigma d\theta_x$

(-199.7)(12)(347)(10)⁻⁶ = -8 (10)⁻⁶
(-199.7)(40)(1042)(10)⁻⁶ = -8327 (10)⁻⁶
(2/3)(300)(40)(1042)(10)⁻⁶ = 8336 (10)⁻⁶
(-41.5)(12)(347)(10)⁻⁶ = -2 (10)⁻⁶
Total = -1 (10)⁻⁶

$\Sigma d\theta_y$

(-0.125)(12)(1042)(10)⁻⁶ = -1563 (10)⁻⁶
(-0.125)(0.5)(25)(347)(60)⁻⁶ = -5 (10)⁻⁶
(0.125)(0.5)(25)(347)(60)⁻⁶ = 5 (10)⁻⁶
(0.125)(12)(1042)(10)⁻⁶ = 1563 (10)⁻⁶
Total = 0

B_z

$\Sigma z_{cx}\,d\theta_y$	$\Sigma y_{cx}\,d\theta_z$
-3 (10)⁻⁶	-790 (10)⁻⁶
0	-5924 (10)⁻⁶
0	7501 (10)⁻⁶
-3 (10)⁻⁶	-790 (10)⁻⁶
Total: -6 (10)⁻⁶	-3 (10)⁻⁶

$B_x = -6(10)^{-6} - (-3(10)^{-6}) = -3(10)^{-6}$

B_x

$\Sigma z_{cz}\,d\theta_y$	$\Sigma y_{cz}\,d\theta_z$
2 (10)⁻⁶	- 6 (10)⁻⁶
0	-9992 (10)⁻⁶
0	10003 (10)⁻⁶
2 (10)⁻⁶	- 2 (10)⁻⁶
4 (10)⁻⁶	3 (10)⁻⁶

$B_z = 4(10)^{-6} - 3(10)^{-6} = 1(10)^{-6}$

Fig. 5—Analysis of a skewed hinged frame of constant section

involving the relatively small values of da_{xx} and da_{yy} will make practically no difference in the results. It is of particular interest to note that the ratio v_z/v_x at the supports is very nearly equal to the tangent of the skew angle (4/3 in this case). This relationship has been pointed out elsewhere* and is used in various approximate solutions.

Variable cross section

The analysis of a hinged, skewed rigid frame with variable cross section is shown in Fig. 6. This structure, which has been analyzed elsewhere† by means of a classical procedure, is subjected to a one-kip gravity load placed 7 ft from the crown measured along the x axis. The moments assumed are those consistent with the structure simply supported at ends A and B. In this example, as well as in all succeeding examples, no corrections were added to such terms as y^2da_{xx} in computing analogous moments of inertia and products of inertia, such corrections being negligible. A check of the geometry, similar to that shown in Fig. 5, was made for this and all other examples, and in all cases the check was conclusive. Such a check has been included, for illustrative purposes, in connection with one subsequent example—the unsymmetrical, hingeless skewed arch of Fig. 7.

Unsymmetrical hingeless skewed arch

The skewed structure shown in Fig. 7 was analyzed for the condition of a one-kip gravity load placed 25 ft from the left springing, measured along the x axis. This analysis is shown in Fig. 7 and 8. The assumed curves of moment are those which correspond to the structure considered cantilevered from the left support. In this and the following example (Fig. 10) the bending stiffnesses, K_{n1} and K_{n2}, and the torsional stiffness, K_t, are relative values based on a ratio E/G equal to 2.5. The numerical values of these relative stiffnesses, shown tabulated in the lower right-hand side of Fig. 7, were determined by assuming E equal to 2.5 and G equal to 1.0. As a result, all the da values used are relative, but a study of the analytical procedure will show this has no effect on the results. It is of interest, in this unsymmetrical case, to observe that the ratio v_z/v_x at the supports differs rather appreciably from the value of the tangent of the skew angle. A numerical check of the geometry is shown in Fig. 9. The relative values of da were used.

Symmetrical skewed arch

An analysis of a symmetrical skewed arch, the properties of whose segments are identical to those of segments 1 through 10 of the arch of Fig. 7, is shown in Fig. 10. The gravity load of one kip is placed at the quarter point, and the assumed moment curves are those corresponding to the structure considered cantilevered from the left end.

Lateral loading and temperature change

In Figs. 11a and 11b, respectively, the structure shown in Fig. 10 is analyzed for the effect of a lateral load, in the z direction, applied to the face of the structure, and for the effect of a temperature rise of 40 F. Since

*Rathbun, *op. cit.*
†Weiner, *op. cit.*

Section	Length	$1/K_{xx}$	$1/K_{yy}$	$1/K_{zz}$	$1/K_{xy}$	$dα_{xx}$	$dα_{yy}$	$dα_{zz}$	$dα_{xy}$	x	y	z	$y\,dα_{zz}$	$y\,dα_{xx}$	$x\,dα_{xy}$	$z\,dα_{xy}$	$y^2dα_{xx}$	$x^2dα_{yy}$	$xy\,dα_{xy}$	$z^2dα_{yy}$	$y^2dα_{zz}$	$x^2dα_{zz}$	$z^2dα_{xx}$	$yz\,dα_{xy}$	$zx\,dα_{yy}$
Hinge		0	0	∞	0	0	0	∞	0	-2060	0	-2455	0	0	0	0	0	0	0	0	0	∞	0	0	0
1	3.5	1 279	13 905	20 071	-4 138	0448	4867	7025	-1 448	-2110	176	-2515	1 236	0079	3 055	3 642	0 001	2 167	0 054	3 078	0 022			0 064	2 583
2	3.5	0 218	43 424	58 510	-2 608	0076	15 198	20 479	-0 913	-2167	5 20	-2583	10 649	0040	1 978	2 358	0 002	7 137	0 103	10 140	0 554			0 123	8 507
3	3.5	0 142	25 260	34 017	-1 516	0050	8 841	11 906	-0 531	-2188	8 70	-2608	10 358	0044	1 162	1 385	0 004	4 232	0 101	6 013	0 901			0 120	5 045
4	3.5	0 104	16 377	22 144	-0 982	0036	5 732	7 750	-0 344	-2210	12 20	-2634	9 455	0044	0 760	0 906	0 005	2 800	0 093	3 977	1 154			0 110	3 337
5	3.5	2 109	5 049	9 701	3226	0738	1 767	3 395	1 129	-2205	15 72	-2628	5 337	1 160	-2 489	- 2 967	0 182	0 859	-0 392	1 221	0 839			-0 466	1 024
6	3.5	5624	0639	8128	1 843	1 968	0224	2 845	0645	-1892	17 40	-2255	4950	3424	-1 220	- 1 454	0 596	0 080	-0 212	0 114	0861			-0 253	0 095
7	3.5	11 627	0880	16 944	3119	4 069	0 308	5 930	1 092	-1560	18 40	-1859	10 911	7487	-1 704	- 2 030	1 378	0 075	-0 313	0 106	2 008			-0 373	0 389
8	3.5	26 888	1 229	37 361	5 619	9 411	0 430	13 076	1 967	-1220	19 22	-1454	25 132	18 088	-2 400	- 2 860	3 476	0 064	-0 461	0 091	4 830			-0 550	0 076
9	3.5	61 402	1 443	84 474	9 181	21 491	0 505	29 566	3 213	-876	19 85	-1044	58 689	42 660	-2 815	- 3 354	8 468	0 039	-0 559	0 055	11 649			-0 666	0 046
10	3.5	135 925	1 172	184 468	12 132	47 574	0 410	64 564	4 246	- 527	20 30	- 628	131 065	96 575	-2 238	- 2 666	19 605	0 011	-0 454	0 016	26 606			-0 541	0 014
11	3.5	202 306	0 305	272 108	6 471	70 807	0 107	95 238	2 265	- 178	20 45	- 212	194 762	144 800	-0 403	- 0 480	29 612	0	-0 083	0 001	39 829			-0 098	0 001
						156 668	38 389						462 544	314 401	-6 314	- 7 520	63 329	17 464	- 2 123	24 812	89 253			- 2 530	20 817
						×2•	×2•						×2•	×2•	×2•	×2•	×2•	×2•	×2•	×2•	×2•			×2•	×2•
					Totals	313 336	76 778	∞					925 088	628 802	-12 628	-15 040	126 658	34 928	- 4 246	48 624	178 506	∞		- 5 060	41 634
														641 430		-15 040	15 040								
					Subtract:												131 307	0	0	0	0	0	0	-3 079	0
																	- 4 649	34 928	- 4 246	49 624	178 506	∞		- 1 981	41 634
																	38 771			228 130		∞		- 43 615	

Centroids

$\bar{x}_x = 0$ $\quad \bar{y} = 0$

$\bar{y}_x = 0$ $\quad \bar{z} = \dfrac{641\,430 \times (10)^6}{313\,336 \times (10)^6} = 20.471$ ft

$\bar{x}_y = 0$ $\quad \bar{z}_y = 0$

♦ These signs are opposite in other half of structure

Section	m^*_{z0}	m^*_{z0}	$m_{z0}dα_{xx}$ $(10)^6$	$m_{z0}dα_{xy}$ $(10)^6$	$m_{z0}dα_{zz}$ $(10)^6$	$zm_{z0}dα_{xx}$ $(10)^6$	$ym_{z0}dα_{zz}$	$xm_{z0}dα_{zz}$	$zm_{z0}dα_{xx}$	$ym_{z0}dα_{xx}$	$xm_{z0}dα_{xy}$
Hinge A	0	0	0	0	* 0	0	0	0	0	0	0
1	- 0402	0335	- 0018	0058	0235	- 0146	0041			- 0003	- 0123
2	- 0858	0717	- 0007	0078	1468	- 0202	0763			- 0003	- 0170
3	- 1025	0858	- 0005	0054	1021	- 0142	0888			- 0004	- 0119
4	- 1199	1005	- 0004	0041	0779	- 0109	0950			- 0005	- 0091
5	- 1159	0971	- 0086	- 0131	0330	0344	0519			- 0135	0289
6	1340	- 1125	0264	0086	- 0320	- 0195	- 0557			0459	- 0163
7	3993	- 3350	1625	0436	- 1986	- 0810	- 3655			2990	- 0680
8	6706	- 5627	6311	1319	- 7358	- 1918	- 14143			12129	- 1609
9	9452	- 7932	20313	3037	- 23450	- 3171	- 46549			40323	- 2661
10	10177	- 8540	48416	4322	- 55135	- 2714	- 111925			98284	- 2277
11	8804	- 7388	62 338	1994	- 70356	- 0423	- 143882			127480	- 0355
12	7404	- 6212	52426	- 1677	- 59167	- 0355	- 120996			107200	- 0298
13	6031	- 5060	28692	- 2561	- 32672	- 1608	- 66324			58243	- 1349
14	4658	- 3908	10011	- 1497	- 11555	- 1562	- 22937			19869	- 1311
15	3304	- 2773	3109	- 0650	- 3626	- 0945	- 6969			5977	- 0793
16	1967	- 1651	0800	- 0215	- 0979	- 0399	- 1801			1473	- 0335
17	0660	- 0555	0130	- 0042	- 0158	- 0096	- 0274			0226	- 0081
18	- 0571	0479	- 0042	0064	0162	0169	0255			- 0066	0142
19	- 0591	0495	- 0002	- 0020	0384	- 0053	0468			- 0003	- 0045
20	- 0505	0423	- 0003	- 0027	0503	- 0070	0438			- 0002	- 0059
21	- 0422	0353	- 0003	- 0038	0723	- 0100	0376			- 0002	- 0084
22	- 0198	0165	- 0009	- 0029	0116	- 0072	0021			- 0001	- 0060
Hinge B	0	0	0	0	0	0	0	0	0	0	0
			234256	4 602	-26043	-14 577	-535293			474429	-12 232
					Subtract:	0	0	0	0	479545	0
						-14 577	-535293			- 5116	-12 232
						520 716				7 116	
					Subtract:	- 8005				-99553	
						528 721				106 669	

Since $\bar{y}_x$ equals infinity these values are not necessary

* These values were obtained by assuming the frame to be simply supported at ends A and B

$J'_1 = 228\,130 - 49\,064 \times 179\,066$

$J'_2 = \infty$

$A'_2 = 313\,336 \times (10)^6 \times 0.126 \times (10)^6 \times 313\,210 \times (10)^9$

$A'_1 = 76\,778 \times (10)^9$

$A'_3 = \infty$

$B'_2 = 228\,130 \times (10)^6 \times 0.072 \times (10)^6 \times 228\,058 \times (10)^6$

$B'_1 = \infty$

$B'_3 = 38\,771 \times (10)^6$

$J''_2 = 228\,058 \times (10)^6 - 49\,064 \times (10)^6 \times 178\,994 \times (10)^6$

$J''_1 = \infty$

$J''_3 = 38\,771 \times (10)^6 - 8\,341 \times (10)^6 \times 30\,430 \times (10)^6$

$F_x = -44\,408 \times (10)^9$

$F_y = 0$

$T_x = -11\,244 \times (10)^7$

$T_y = 0\,901 \times (10)^7$

$P'_x = 234\,256 \times (10)^6 \times 4\,441 \times (10)^6 \times 238\,697 \times (10)^9$

$P'_y = 4\,602 \times (10)^9$

$P'_z = -261\,043 \times (10)^9$

$M'_x = 528\,721 \times (10)^7 + 11\,244 \times (10)^7 \times 539\,965 \times (10)^7$

$M'_y = 0$

$M'_z = 7\,116 \times (10)^7 \times 99\,584 \times (10)^6 \times 2\,150 \times (10)^6 \times 108\,850 \times (10)^7$

$v_{x1} = \dfrac{M'_z}{J'_1} = \dfrac{539\,965 \times (10)^7}{178\,994 \times (10)^6} = 0.302^k$

$v_{y1} = \dfrac{M'_z}{J'_2} \times 0$

$v_{z1} = \dfrac{M'_z}{J'_3} = \dfrac{108\,850 \times (10)^7}{30\,430 \times (10)^6} = 0.358^k$

Sect	K_{xx} ×(10)	K_{yy} ×(10)	K_z ×(10)
1	232049	498	660
2	162434	171	229
3	194622	294	394
4	224564	452	608
5	295676	1031	1404
6	313641	1230	1606
7	245523	590	803
8	188633	268	356
9	143721	118	159
10	110785	054	073
11	97311	037	049

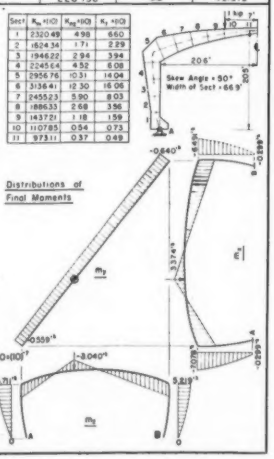


Fig. 6—Analysis of a skewed hinged frame of variable section

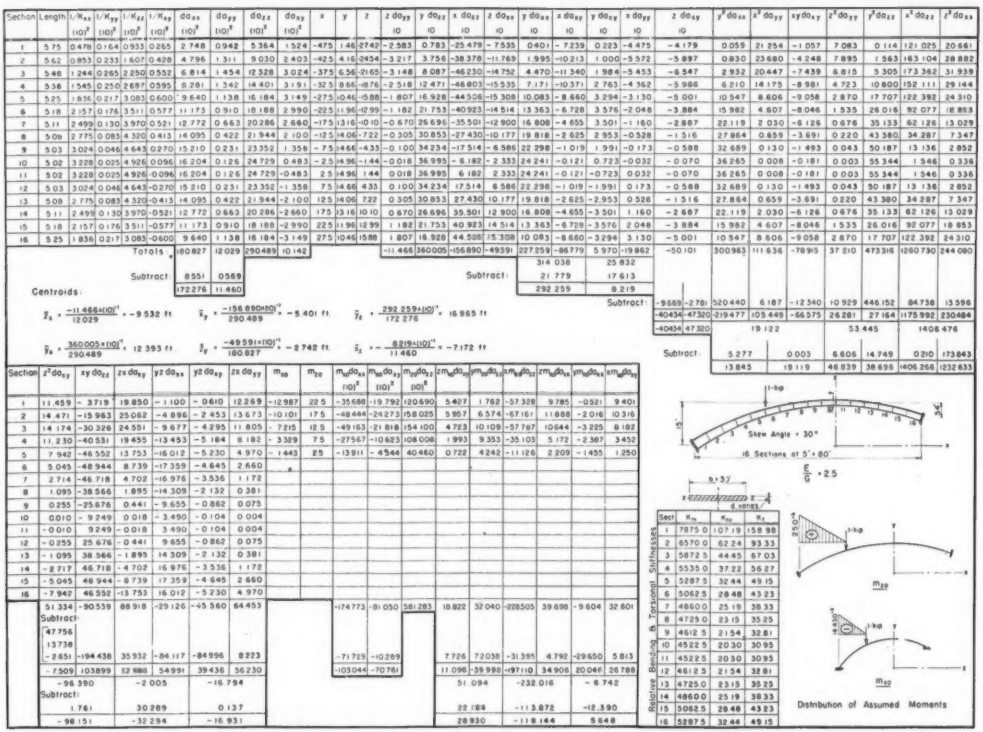


Fig. 7—Analysis of an unsymmetrical, hingeless skewed arch. Part 1

$J_x' = 53.445 - 6.728 - 14.972 + 31.845$ $\qquad F_x = -3.673$ $\qquad \bar{x} = 2.418$

$J_y' = 1406.476 - 4.677 - 244.490 + 1157.309$ $\qquad F_y = -0.483$ $\qquad \bar{y} = -29.218$

	A_x = 1.808	A_y = 0.120	A_z = 2.905	A_{xy} = 0.101	J_x = 53.445	J_y = 1406.476	J_z = 19.122	J_{xy} = -86.390	J_{yz} = -2.005	J_{zx} = -16.794	F_x = -1.748	F_y = -0.611	β_z = 5.813	M_x = 51.094	M_y = -232.016	M_z = -6.742
Subtract:	0.513	0.019		-0.042	9.467	194.712	0	9.346	0	0	-1.148	-0.193		26.569	-165.541	-14.688
	1.295	0.101		0.143	43.978	1211.764	19.122	-105.736	-2.005	-16.794	-0.600	-0.618		24.525	-66.475	7.946
Subtract:	0.202	0.016			9.226 14.743	0.210 284.336	6.416 0.003	1.761	40.396	0.175	[-3.673] +0.684	[-0.483] +0.406		[+2.418] -2.592	[-29.218] +8.699	[+1.181] -1.294
	1.093	0.085	2.905		34.752 29.209	1211.554 957.428	12.706 19.119	-107.497	-42.401	-16.969	2.389	-0.541	5.813	24.699	-46.156	8.059

$J_x' = 43.978 - 9.382 - 14.906 + 19.671$

$J_y' = 1211.764 - 6.691 - 389.137 + 815.936$

$J_z' = 19.122 + 8.205 - 0.089 + 10.828$

$v_{xi} = \frac{M_y'}{J_y'} = \frac{24.699}{19.671} = 1.256$ k $\qquad v_x = 0 - 1.256 = -1.256$ k

$v_{yi} = \frac{M_x'}{J_y'} = \frac{-46.156}{815.936} = -0.057$ k $\qquad v_y = -1.000 - (-0.057) = -0.943$ k

$v_{zi} = \frac{M_z'}{J_z'} = \frac{+8.059}{10.828} = 0.744$ k $\qquad v_z = 0 - 0.744 = -0.744$ k

at A

$m_{xi} = \frac{2.389}{1.093} + (0.744)(-18.965) - (-0.057)(-26.128) = -11.933$'k $\qquad m_x = -14.430 - (-11.933) = -2.497$'k

$m_{yi} = \frac{-0.541}{0.085} + (1.256)(-19.338) - (0.744)(-42.828) = 1.277$'k $\qquad m_y = 0 - 1.277 = -1.277$'k

$m_{zi} = \frac{5.813}{2.905} + (-0.057)(-44.599) - (1.256)(-12.393) = 20.077$'k $\qquad m_z = 25.000 - 20.077 = 4.923$'k

at Load

$m_{xi} = \frac{2.389}{1.093} + (0.744)(-5.465) - (-0.057)(-11.693) = -2.547$'k $\qquad m_x = 0 - (-2.547) = 2.547$'k

$m_{yi} = \frac{-0.541}{0.085} + (1.256)(-4.903) - (0.744)(-17.828) = 0.741$'k $\qquad m_y = 0 - 0.741 = -0.741$'k

$m_{zi} = \frac{5.813}{2.905} + (-0.057)(-19.599) - (1.256)(-0.893) = 4.240$'k $\qquad m_z = 0 - 4.240 = -4.240$'k

at B

$m_{xi} = \frac{2.389}{1.093} + (0.744)(-7.365) - (-0.057)(20.062) = -2.185$'k $\qquad m_x = 0 - (-2.185) = 2.185$'k

$m_{yi} = \frac{-0.541}{0.085} + (1.256)(26.852) - (0.744)(37.172) = -0.342$'k $\qquad m_y = 0 - (-0.342) = 0.342$'k

$m_{zi} = \frac{5.813}{2.905} + (-0.057)(35.401) - (1.256)(-2.793) = 3.526$'k $\qquad m_z = 0 - 3.526 = -3.526$'k

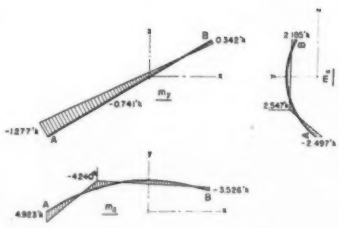


Fig. 8—Analysis of an unsymmetrical, hingeless skewed arch. Part 2

Section	Moments			Angle changes, $d\Theta \times 10^2$			Displacements, $\delta \times 10^2$		
	m_x	m_y	m_z	$d\Theta_x = m_x\,da_{xx} + m_y\,da_{xy}$	$d\Theta_y = m_y\,da_{yy} + m_x\,da_{xy}$	$d\Theta_z = m_z\,da_{zz}$	$\delta_x = z_{cz}\,d\Theta_y - y_{cz}\,d\Theta_z$	$\delta_y = x_{cy}\,d\Theta_z - z_{cy}\,d\Theta_x$	$\delta_z = y_{cz}\,d\Theta_x - x_{cz}\,d\Theta_y$
1	−2.224	−1.235	4.398	−7.994	−4.552	23.591	9.038	−218.595	−22.789
2	−1.513	−1.125	3.072	−9.959	−5.111	27.740	1.002	−242.803	−25.865
3	−0.580	−1.028	1.369	−7.061	−3.249	16.877	−4.037	−141.437	−16.816
4	1.578	−0.930	−0.711	10.099	3.787	−10.239	1.763	100.431	21.054
5	1.960	−0.821	−3.169	16.309	5.238	−51.287	45.328	349.046	31.464
6	2.122	−0.723	−3.503	21.547	5.687	−63.713	68.814	399.802	38.566
7	1.065	−0.623	−1.715	11.945	2.420	−34.790	43.340	198.006	19.967
8	0.232	−0.516	−0.304	2.186	0.259	−6.671	9.185	33.716	3.410
9	−0.377	−0.419	0.731	−6.303	−0.609	17.070	−24.761	−77.659	−9.697
10	−0.763	−0.322	1.388	−12.520	−0.410	34.324	−51.290	−135.041	−18.833
11	−0.924	−0.212	1.668	−14.870	0.419	41.248	−61.647	−137.046	−22.351
12	−0.862	−0.115	1.571	−12.955	1.144	36.686	−53.287	−99.373	−19.850
13	−0.577	−0.017	1.096	−8.097	1.205	24.051	−32.946	−50.276	−12.890
14	−0.066	0.093	0.245	−1.090	0.238	4.970	−6.301	−7.000	−1.851
15	0.667	0.190	−0.983	6.884	−1.821	−17.879	19.018	16.390	12.330
16	1.624	0.287	−2.589	14.751	−4.787	−41.900	36.228	12.599	28.594
				$\Sigma\,d\Theta_x = +2.872 \times 10^{-2}$	$\Sigma\,d\Theta_y = -0.132 \times 10^{-2}$	$\Sigma\,d\Theta_z = +0.078 \times 10^{-2}$	$\delta_x = -0.553 \times 10^{-2}$	$\delta_y = +0.760 \times 10^{-2}$	$\delta_z = +4.443 \times 10^{-2}$

Fig. 9—Geometrical check of solution for the unsymmetrical skew arch

the properties of this structure are shown in Fig. 10, they are not repeated in Fig. 11. In Fig. 11a, which illustrates the analysis for lateral load, the assumed moment curves are those for the structure considered cantilevered from the left support. In Fig. 11b, which illustrates the analysis for temperature change, there are no assumed moments and consequently no analogous loads. The displacements which would occur along the rectangular axes, if the structures were free to expand, are entered as values of the analogous moments M_x and M_z. Because of symmetry M_y is equal to zero. Attention is called to the fact that, since this is an example involving movements, it is necessary to introduce a correction factor because the elastic properties used in the analysis are relative. In this case, with E taken as 432,000,000 psf, the correction factor is 432×10^3 to get results in terms of kips. The effects of rib shortening can be determined in a similar manner.

Influence lines

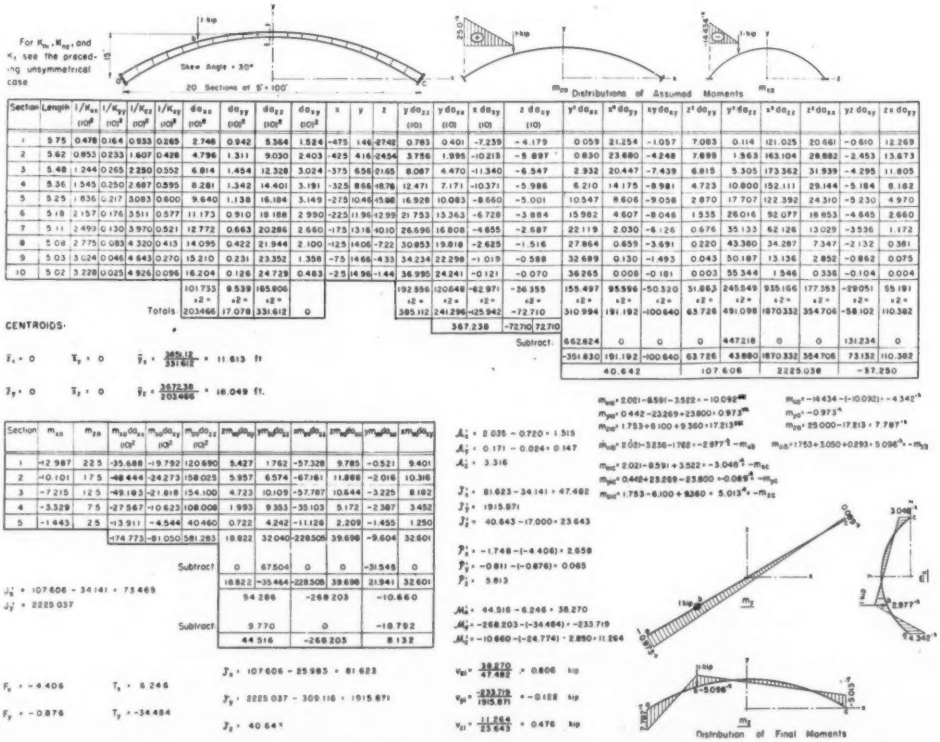
If it is desired to obtain influence lines by making use of Müller-Breslau's concept, the P and M terms will represent the corresponding unit rotations and unit displacements applied to the structure.

CONCLUDING REMARKS

A numerical procedure has been presented for the analysis of skewed rigid frames and arches subjected to loads or deformations in any direction. The procedure is general and is applicable to any skewed arch or frame, whether symmetrical or not, of one span. The results are exact if the relationships between statics and geometry as defined herein, and generally proposed and used by others, are acceptable.

ACKNOWLEDGMENT

The examples are taken from an MS thesis by Dale L. Ferguson, prepared under the writer's direction at Iowa State College, Ames, Iowa.



Section	Length	I/K_{xx} (10)^6	I/K_{yy} (10)^6	I/K_{xy} (10)^6	I/K_{xy} (10)^6	$dσ_{xx}$	$dσ_{yy}$ (10)^6	$dσ_{xx}$ (10)^6	$dσ_{xy}$ (10)^6	x	y	z	$y\,dσ_{xx}$ (10)	$x\,dσ_{yy}$ (10)	$x\,dσ_{xy}$ (10)	$z\,dσ_{xy}$ (10)	$y^2dσ_{xx}$	$x^2dσ_{yy}$	$xydσ_{xy}$	$z^2dσ_{yy}$	$y^2dσ_{xx}$	$x^2dσ_{xx}$	$z^2dσ_{xx}$	$yz\,dσ_{xy}$	$zx\,dσ_{xy}$
1	5.75	0.478	0.164	0.933	0.265	2.748	0.942	5.364	1.524	-475	146	-2742	0.783	0.401	-7.239	-4.179	0.059	21.254	-1.057	7.083	0.114	121.025	20.461	-0.610	12.269
2	5.82	0.853	0.233	1.607	0.428	4.796	1.311	9.030	2.403	-425	416	-2454	3.756	1.995	-10.213	-5.897	0.830	23.680	-4.248	7.899	1.363	163.104	28.882	-2.453	13.673
3	5.48	1.244	0.265	2.230	0.552	6.814	1.454	12.328	3.024	-375	656	2165	8.087	4.470	-11.340	-6.547	2.932	20.447	-7.439	6.815	5.305	173.362	31.939	-4.295	11.805
4	5.36	1.545	0.250	2.687	0.595	8.281	1.342	14.401	3.191	-325	866	1876	12.471	7.171	-10.371	-5.986	6.210	14.175	-8.981	4.723	10.800	152.111	29.144	-5.184	8.182
5	5.25	1.836	0.217	3.083	0.600	9.640	1.138	16.184	3.149	-275	1046	1588	16.928	10.083	-9.660	-5.001	10.547	8.606	-9.038	2.870	17.707	122.392	24.310	-5.230	4.970
6	5.18	2.157	0.176	3.511	0.577	11.173	0.910	18.188	2.990	-225	1196	1298	21.753	13.363	-6.728	-3.884	15.982	4.607	-8.046	1.535	26.016	92.077	18.853	-4.645	2.660
7	5.11	2.493	0.130	3.970	0.521	12.772	0.663	20.286	2.660	-175	1316	1010	26.596	16.808	-4.655	-2.687	22.119	2.030	-6.126	0.676	35.133	62.126	13.029	-3.536	1.172
8	5.08	2.775	0.083	4.320	0.413	14.095	0.422	21.944	2.100	-125	1406	722	30.853	19.818	-2.625	-1.516	27.864	0.659	-3.691	0.220	43.380	34.287	7.347	-2.132	0.381
9	5.03	3.024	0.046	4.643	0.270	15.210	0.231	23.352	1.358	-75	1466	433	34.234	22.298	-1.019	-0.588	32.689	0.130	-1.493	0.043	50.187	13.136	2.852	-0.862	0.075
10	5.02	3.228	0.025	4.926	0.096	16.204	0.126	24.729	0.483	-25	1496	144	36.995	24.241	-0.121	-0.070	36.265	0.008	-0.181	0.003	55.344	1.546	0.336	-0.104	0.004
						101.733 ±Σ =	9.539 ±Σ =	165.906 ±Σ =					192.996 ±Σ =	120.648 ±Σ =	-62.971 ±Σ =	-36.355 ±Σ =	155.497 ±Σ =	95.596 ±Σ =	-50.520 ±Σ =	31.863 ±Σ =	245.549 ±Σ =	935.166 ±Σ =	177.353 ±Σ =	-29.051 ±Σ =	55.191 ±Σ =
					Totals	203.466	17.078	331.612	0				385.112	241.296	-125.942	-72.710	310.994	191.192	-100.640	63.726	491.098	1870.332	354.706	-58.102	110.382
														367.238		-72.710	72.710								
																Subtract	662.824	0	0	0	447.218	0	0	131.234	0
																	-351.830	191.192	-100.640	63.726	43.880	1870.332	354.706	-73.132	110.382
																	40.642			107.606		2225.038		-37.250	

$\bar{x}_x = 0 \qquad \bar{y}_x = 0 \qquad \bar{y}_z = \frac{385.12}{331.612} = 11.613$ ft

$\bar{y}_y = 0 \qquad \bar{x}_z = 0 \qquad \bar{y}_z = \frac{3672.38}{203.466} = 18.049$ ft.

Section	m_{xo}	m_{yo}	$m_{xo}dσ_{xx}$ (10)^2	$m_{xo}dσ_{xy}$ (10)^2	$m_{yo}dσ_{xy}$ (10)^2	$zm_{xo}dσ_{xx}$	$ym_{xo}dσ_{xx}$	$zm_{yo}dσ_{xy}$	$ym_{yo}dσ_{xy}$	$zm_{xo}dσ_{xy}$	$xm_{xo}dσ_{xy}$
1	+12.987	22.5	-35.688	-19.792	120.690	5.427	1.762	-57.328	9.785	-0.521	9.401
2	+10.101	17.5	-48.444	-24.273	158.025	5.957	6.574	-67.161	11.888	-2.016	10.316
3	+7.215	12.5	-49.183	-21.818	154.100	4.723	10.109	-57.787	10.644	-3.225	8.182
4	+3.329	7.5	-27.567	-10.623	108.008	1.993	9.353	-35.103	9.172	-2.387	3.452
5	+1.443	2.5	+13.911	-4.544	40.460	0.722	4.242	-11.128	2.209	-1.455	1.250
			-174.773	-81.050	581.283	18.822	32.040	-228.505	39.698	-9.604	32.601
					Subtract	0	67.504	0	0	-31.545	0
						18.822	-35.464	-228.505	39.698	21.941	32.601
						54.286		-268.203		-10.660	
					Subtract	9.770		0		-18.792	
						44.516		-268.203		8.132	

$\bar{J}_x' = 107.606 - 34.141 + 75.469$

$\bar{J}_y' = 2225.037$

$J_x = 107.606 - 25.985 = 81.623$

$J_y = 2225.037 - 309.116 = 1915.871$

$J_z = 40.643$

$F_x = -4.406 \qquad T_x = 6.246$

$F_y = -0.876 \qquad T_y = -34.484$

$\bar{A}_x' = 2.035 - 0.720 + 1.315$

$\bar{A}_y' = 0.171 - 0.024 + 0.147$

$\bar{A}_z' = 3.316$

$m_{xx} = 2.021 - 8.591 - 3.522 = -10.092^{m_{xx}}$

$m_{yx} = 0.442 - 23.269 + 23.800 = 0.973^{m_{yx}}$

$m_{zx} = 1.753 + 6.100 + 9.360 = 17.213^{m_{zx}}$

$m_{xy} = -16.434 - (-10.092) = -6.342^{m_{xy}}$

$m_{yy} = -0.973^{m_{yy}}$

$m_{zy} = 25.000 - 17.213 = 7.787^{m_{zy}}$

$m_{xo} = 2.021 - 8.591 + 3.522 = -3.048^{m_{xo}} - m_{xc}$

$m_{yo} = 0.442 + 23.269 - 23.800 = -0.089^{m_{yo}} - m_{yc}$

$m_{zo} = 1.753 - 6.100 + 8.360 = 5.013^{m_{zo}} - m_{zc}$

$m_{xo} = 1.753 + 3.050 + 0.293 = 5.096^{m_{xo}} - m_{xb}$

$M_x' = 44.516 - 6.246 + 38.270$

$M_y' = -268.203 - (-34.484) = -233.719$

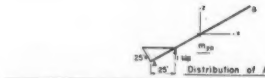
$M_z' = -10.660 - (-24.774) = 2.850 + 11.264$

$v_{xx} = \frac{38.270}{47.482} = 0.806$ kip

$v_{yx} = \frac{-233.719}{1915.871} = -0.122$ kip

$v_{zx} = \frac{11.264}{23.643} = 0.476$ kip

Fig. 10—Analysis of a skewed arch subjected to a gravity load



Distribution of Assumed Moments

Section	m_{ya}	m_{yb}	$m_{ya}dx_{ya}$ $(10)^3$	$m_{yb}dx_{yb}$ $(10)^3$	$m_{ya}dx_{ya}$ $(10)^3$	$m_{yb}dx_{yb}$ $(10)^3$	$zm_{ya}dx_{ya}$	$zm_{yb}dx_{yb}$	$zm_{ya}dx_{ya}$	$zm_{yb}dx_{yb}$	$xm_{ya}dx_{ya}$	$xm_{yb}dx_{yb}$	$xm_{ya}dx_{ya}$	$xm_{yb}dx_{yb}$
1	-979	225	-26.903	34.290	+4.920	21.195	4.091	-5.812	7.377	-9.402	-0.393	0.501	7.087	-10.068
2	-709	175	-34.004	42.052	+17.037	22.943	4.161	-8.630	8.345	-10.320	-1.415	1.749	7.241	-9.751
3	-469	125	-31.938	37.800	+14.183	18.175	3.071	-3.935	6.919	-8.184	-2.096	2.480	5.319	-6.816
4	-259	75	-21.448	23.933	-8.265	10.065	1.550	-1.888	4.023	-4.490	-1.857	2.073	2.686	-3.271
5	-069	25	-6.632	7.873	-2.173	2.845	0.385	-0.452	1.056	-1.250	-0.696	0.823	0.597	-0.782
			+120.965	+145.948	-56.578	75.223	13.238	-17.717	27.720	-33.646	-6.457	7.626	22.930	-30.688
			24.983		18.645		-4.479		-5.925		1.169		-7.758	
						Subtract:	0		0		4.509		0	
							-6.479		-5.925		-3.340		-7.758	
							-6.479		**5.925**		4.418			
						Subtract:	-4.038		0		1.546			
							-0.441		5.925		2.872			

$F_x = (-7.271)\frac{(-0.441)}{73.485} = 0.0435$ ←

$F_y = 7.271\frac{5.925}{2225.037} = 0.0194$ ↓

$T_x = (-7.271)\frac{0.250}{2.035} = -0.8932$

$T_y = 7.271\frac{0.186}{0.171} = 7.9088$

$\mathcal{F}'_x = 0.250 - 0.0435 = 0.2065$

$\mathcal{F}'_y = 0.186 - 0.0194 = 0.1676$

$\mathcal{F}'_z = 0$

at A

$m_{xa} = 0.1568 - 2.1179 - 0.0300 = -1.9911$'k
$m_{ya} = 1.1401 - 0.2737 + 5.8670 + 6.7334$'k
$m_{za} = 0 + 0.0520 + 0.1100 + 0.1620$'k

$m_x = -11.25 + 1.991 = -9.259$'k
$m_y = 25.0 - 6.733 + 18.267$'k
$m_z = 0 - 0.162 = -0.162$'k

at Load

$m_{xa} = 0.1568 - 0.7577 + 0.0150 = -0.6259$'k
$m_{ya} = 1.1401 - 0.1368 + 2.9335 + 3.9368$'k
$m_{za} = 0 + 0.0260 + 0.0034 + 0.0294$'k

$m_x = 0 + 0.626 = 0.626$'k
$m_y = 0 - 3.937 = -3.937$'k
$m_z = 0 - 0.029 = -0.029$'k

at B

$m_{xa} = 0.1568 - 2.1179 + 0.0300 = -1.9311$'k
$m_{ya} = 1.1401 + 0.2737 - 3.8670 = -4.4532$'k
$m_{za} = 0 - 0.0520 + 0.1100 = 0.0580$'k

$m_x = 0 - (-1.938) = 1.931$'k
$m_y = 0 - (-4.453) = 4.453$'k
$m_z = 0 - 0.058 = -0.058$'k

$\mathcal{M}'_x = -0.441 - (-0.893) = 0.452$

$\mathcal{M}'_y = 5.925 - 7.909 = -1.984$

$\mathcal{M}'_z = 4.417 - 2.038 - (-0.4063) = 2.785$'

$v_{x1} = \frac{0.452}{47.482} = 0.0095$ h

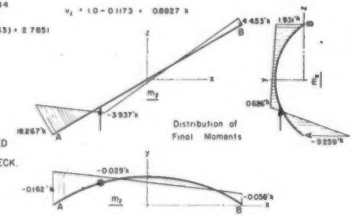
$v_{y1} = \frac{-1.984}{1915.871} = -0.0010$ h

$v_{z1} = \frac{2.785}{23.643} = 0.1173$ h

$v_x = 0 - 0.0095 = -0.0095$ h

$v_y = 0 - (-0.0010) = 0.0010$ h

$v_z = 1.0 - 0.1173 = 0.8827$ h



(a) LATERAL LOAD APPLIED TO THE FACE OF THE DECK.

Temperature rise equals 40° F.

C_T (coef. of expansion) = 0.000006

Total expansion:

x direction = 0.0240 ft.

z direction = 0.0139 ft.

	M_x = 0.0240	M_y = 0	M_z = 0.0139
Subtract:	- 0.0127	0	- 0.0083
	0.0367	0	0.0222

$F_x = (-7.271)\frac{0.0367}{73.485} = -0.00363$

$\mathcal{F}'_x = 0 - (-0.00363) = 0.00363$

$\mathcal{M}'_x = 0.0367$

$\mathcal{M}'_y = 0$

$\mathcal{M}'_z = 0.0139 - 0.0240\frac{37.290}{81.625} = 0.0248$

$v_{x1} = \frac{0.0367}{47.482} 432\ (10)^3 = 334$ h

$v_{y1} = 0$

$v_{z1} = \frac{0.0248}{23.643} 432\ (10)^3 = 454$ h

$m_{xa} = 0 - (-7007) = 7007$'k

$m_{ya} = 0 - 13,060 = -13,060$'k

$m_{za} = 0 - 3878 = -3878$'k

$m_{xa} = 1193 - 6200 = -7007$'k

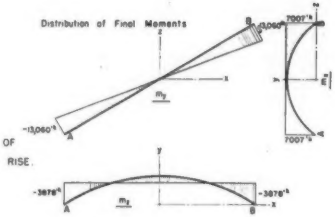
$m_{ya} = -9640 + 22,700 = 13,060$'k

$m_{za} = 3878$'k

$m_{xb} = m_{xa}$

$m_{yb} = -m_{ya}$

$m_{zb} = m_{za}$



(b) EFFECTS OF TEMPERATURE RISE.

Fig. 11—Effects of lateral load and temperature change on arch of Fig. 10

Discussion of a paper by James P. Michalos:

Analysis of Skewed Rigid Frames and Arches*

By MAURICE BARRON and AUTHOR

By MAURICE BARRON†

In presenting a method of analysis for skewed reinforced concrete rigid frames and arches the author presents a two-step procedure which is a special adaptation of his general method presented elsewhere.[1] In discussing the general method the writer pointed out[2] that it could be applied to skewed arches and rigid frame bridges and when so applied would exactly parallel the methods and theory presented by Prof. J. Charles Rathbun.[3]

The Rathbun method is an identical two-step procedure. The first step consists of the stress analysis of a statically determinate structure (redundants removed) and the determination of deflections. The second step of the Rathbun method consists of correcting the above stresses for a system of redundants applied to the residual structure which corrects the errors in geometry and still maintains static equilibrium. The Rathbun equations of elastic equilibrium are exactly the same as the author's equations of geometry. The final stresses are the stresses for the simple structure to which have been added corrective stresses due to the redundants which simultaneously satisfy the geometric deflections of the residual structure.

Continuing the parallel, the second step of both methods solves a system of simultaneous equations. This numerical solution of simultaneous equations for space structures has been shown to be unnecessary.[4] Furthermore, when used, the numerical solution of the simultaneous equations has been shown to frequently yield incorrect answers which cannot be detected easily.[5]

These shortcomings are incorporated in the author's method. The numerical checks which the author shows lead him to a false sense of security. To demonstrate the validity of this statement it is only necessary to introduce a large error (say 200 percent) in the values of K_t, or J_x, J_y, J_z, and to recompute the numerical answers. They will be the same as in the original examples, and the author's check will be just as convincing that the answers are correct. An inspection of the numerical checks (Fig. 5) shows this without the recomputing. Of the four elements which add to zero, two contain numerical values of four significant figures and two are of an order insignificantly smaller (one significant figure).

*ACI JOURNAL, Feb. 1952, *Proc.* V. 48, p. 437. Disc. 48-30 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 24, No. 4, Dec. 1952, Part 2, *Proceedings*, V. 48.

†Consulting Engineer, Farkas & Barron, New York, N. Y.

In presenting Eqs. 13 the author states they are the solution of Eqs. 11 and 12. The solution is not shown and should be further examined by the author for an error. For vertical loads Eqs. 13 are close but they cannot be general equations for all loadings. For instance it is obvious that results due to temperature change using Eqs. 13 will not satisfy the author's Eqs. 14. The same is true for all volume changes and for differential movements of the foundations. The author's analysis is an approximate one which is satisfactory only for vertical loads. Furthermore, the degree of approximation cannot be determined.

Analysis of skewed arches and rigid frame bridges has received considerable attention in recent years. More than one method[6,7,8] will yield results in less than one tenth the time with equal or greater accuracy.

It is unfortunate that more effort is not expended on the design phase which this paper does not cover. Also, it is unfortunate that the American Standard Letter Symbols for Structural Analysis (ASA-Z10.8-1949) was not adopted for this paper. This non-conformity makes the paper more difficult to understand and less likely to be compared with other methods to determine its relative merit.

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3. Rathbun, J. Charles, "Analysis of the Stresses in the Ring of a Concrete Skew Arch," *Transactions*, ASCE, V. 87, 1924, p. 611.

4. Barron, Maurice, "Reinforced Concrete Skewed Rigid-Frame and Arch Bridges," *Transactions*, ASCE, V. 116, 1951, p. 999.

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6. Hayden, Arthur G., and Barron, Maurice, *The Rigid-Frame Bridge*, John Wiley and Sons, Inc., New York, N. Y., 1950.

7. Hodges, Richard M., "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," *Transactions*, ASCE, V. 109, 1944, p. 913.

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AUTHOR'S CLOSURE

The author thanks Mr. Barron for his discussion. He will endeavor to answer the questions and objections raised in that discussion.

Mr. Barron states that the Rathbun method[1] is an identical two-step procedure. Rathbun's procedure can be so classified, but so can virtually all methods of structural analysis of statically indeterminate structures, and the identity stops just about there. In the analysis of all statically indeterminate structures it is necessary to satisfy simultaneously requirements of statics and of geometry. The analyst can first assume a solution which satisfies the requirements of statics and, on the basis of this solution, determine the errors in geometry. Then, as a second step, he can correct the errors in the geometry without disturbing statical equilibrium. Or, as an alternative

to the above sequence, he can assume a solution which satisfies geometry and then correct the resulting errors in statics. Both Rathbun and the author have used the first sequence. This sequence goes back at least to Maxwell and is sometimes referred to as Maxwell's method or the stress method of analysis.

In the paper under discussion purely arithmetical computations yield exactly the same results as a classical solution, such as Rathbun's, in which a rather complicated system of equations of geometry (six in the general case of a skewed frame or arch) is solved.

Obviously there must be points of similarity between Rathbun's equations and those used in derivation of the numerical method of the paper. As stated in the paper, the author has assumed the same relationships between the statics and geometry of an element of the structure as those used by Rathbun. The author campaigns neither for nor against these assumptions. He only states in the paper that the results are exact—and identical to those which would be obtained by Rathbun's method—if the assumptions made by Rathbun are acceptable. These same assumptions are inherent in the simplifications proposed by Weiner,[2] Hodges,[3] and Barron.[4] Given any other relationships between angle change and moment (expressed in the paper by the terms K) the proposed method would be applied exactly as in the present paper.

In view of Mr. Barron's comments it should probably be emphasized that no simultaneous equations are solved by the designer or computer in applying the procedure proposed by the author. Eqs. 13 represent a complete solution for the six redundants, and the designer is concerned only with the arithmetical determination of the various terms of these equations with the aid of the tabular form in Fig. 3 or Fig. 4. Contrary to Mr. Barron's statement, the solution which yielded Eqs. 13 is exact for any loading conditions since the possible effects of loads in all directions are included in Eqs. 11 and 12. A solution for the effects of a lateral load applied to a skewed arch is presented in Fig. 11a.

Mr. Barron is mistaken in stating that results due to temperature change, when obtained through use of Eqs. 13, will not satisfy Eqs. 14. They will if Eqs. 13 are applied correctly as explained in the text and illustrated in Fig. 11b.

In making an analysis for volume changes all $\mathcal{P}'$ terms in Eqs. 13 are set equal to zero. This is because there are no angle changes, and thus no analogous loads, since in the assumed solution the structure is considered to expand or contract freely. As a result, all that part of the tabulation involving angle changes is eliminated. Due to the temperature rise and coefficient of expansion assumed in Fig. 11b, the supports, if allowed to move freely, will move relative to each other, in both the x and z directions, the amounts indicated. These displacements are the analogous moments M_x and M_z. From these values the terms $\mathcal{M}'$ are evaluated, and using terms $\mathcal{A}'$ and $\mathcal{J}'$ from Fig. 10, Eqs. 13 are solved as shown in Fig. 11b.

A check of the solution in Fig. 11b, similar to the check in Fig. 9, does satisfy the requirements of geometry expressed in Eqs. 14.

The author wishes to assure Mr. Barron that the numerical checks do not lead the former to a false sense of security. In the last paragraph under "Numerical examples—Constant cross section," the author states the following. "Certain tabulated values in Fig. 5 can be eliminated with only slight loss in accuracy. These are the relatively small values of da_{xx} and da_{yy} which are dependent upon the large bending stiffness K_{n2} (this, of course, should be K_{n1} in the paper). As might be expected, because of this very large bending resistance, the elimination of all computations involving the relatively small values of da_{xx} and da_{yy} will make practically no difference in the results."

The simplifications outlined above are not necessarily permissible in a skewed arch in which, because of the great variation in slope of the elements, the variation in values of da_{xx} and da_{yy} will not be extreme. Such simplifications can also become objectionable in skewed frames of variable section. In addition, since the ratio of a support reaction in the z direction to that in the x direction is nearly equal to the tangent of the skew angle in symmetrical skewed frames, this relationship is often used in making approximate solutions. However, as pointed out in the paper, this relationship does not hold in the case of an unsymmetrical structure.

In conclusion, the method proposed in the paper is purely arithmetical but will yield the exact results of a classical method. It is applicable to symmetrical or unsymmetrical frames and arches, hinged or fixed at their bases, and subjected to any loading or any deformations. Because of the form of computations it is convenient and easy for the designer to decide in any particular case what computations can be eliminated without appreciable loss in precision. He is not dependent upon arbitrary simplifications which may or may not be permissible in his particular case.

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